CS/MATH111 ASSIGNMENT 1

Problem 1: Let $W(n)$ be the number of times "whatsup" is printed by Algorithm WHATSUP (see below) on input n. Determine the asymptotic value of $W(n)$.

Algorithm WHATSUP $(n : \textbf{integer})$
   for $i \leftarrow 1$ **to** $2n$ **do**
      for $j \leftarrow 1$ **to** $(i+1)^2$ **do**
         print("whatsup")

   Your solution must consist of the following steps:

(a) First express $W(n)$ using summation notation $\sum$.

(b) Next, give a closed-form formula for $W(n)$. (A "closed-form formula" should be a simple arithmetic expression without any summation symbols.)

(c) Finally, give the asymptotic value of $W(n)$ using the Θ-notation.

Include a brief justification for each step.

Note: If you need any summation formulas for this problem, you are allowed to look them up.

Problem 2: Consider a sequence defined recursively as $T_0 = 1$, $T_1 = 2$, and $T_n = T_{n-1} + 3T_{n-2}$ for $n \geq 2$. Prove that $T_n = O(2.4^n)$ and $T_n = \Omega(2.3^n)$.

Hint: First, prove by induction that $\frac{1}{2} \cdot 2.3^n \leq T_n \leq 2.4^n$ for all $n \geq 0$. This is similar to the proof from class for the estimate of Fibonacci numbers.

Problem 3: Give the asymptotic values of the following functions, using the Θ-notation:

(a) $7n^2 + 2n^4 + 3n + 1$

(b) $5/n + \log_3 n + 11\sqrt{n}$

(c) $2n(\log n + n^2) + 3n^4/\log n$

(d) $25n^{12} + 1.1^n + n^3 \log^4 n$

(e) $n^7 2^n + 5 \cdot 3^n$

Justify your answer. (Here, you don't need to give a complete rigorous proof. Give only an informal explanation using asymptotic relations between the functions n^c, $\log n$, and c^n.)

Submission. To submit the homework, you need to upload the pdf file into Gradescope (1 submission per group) and iLearn (each student has to submit individually). Late submissions will not be accepted.

Reminders. Remember that only papers created with LaTeX are accepted.