

## CS/MATH111 ASSIGNMENT 4

**Problem 1:** Give the asymptotic value (using the  $\Theta$ -notation) for the number of letters that will be printed by the algorithms below. In each algorithm the argument  $n$  is a positive integer. Your solution needs to consist of an appropriate recurrence equation and its solution. You also need to give a brief justification for the recurrence.

Part (a)

(i) **Algorithm PRINTAS** ( $n$  : integer)

```
    if  $n < 4$ 
        print("A")
    else
        PRINTAS( $\lceil n/3 \rceil$ )
        PRINTAS( $\lceil n/3 \rceil$ )
        PRINTAS( $\lceil n/3 \rceil$ )
        PRINTAS( $\lceil n/3 \rceil$ )
        PRINTAS( $\lceil n/3 \rceil$ )
        for  $i \leftarrow 1$  to  $4n^2$  do print("A")
```

(ii) **Algorithm PRINTBS** ( $n$  : integer)

```
    if  $n < 2$ 
        print("B")
    else
        for  $j \leftarrow 1$  to 8 do PRINTBS( $\lfloor n/2 \rfloor$ )
        for  $i \leftarrow 1$  to  $10n^3$  do print("B")
```

(iii) **Algorithm PRINTCS** ( $n$  : integer)

```
    if  $n < 3$ 
        print("C")
    else
        PRINTCS( $\lceil n/2 \rceil$ )
        PRINTCS( $\lceil n/2 \rceil$ )
        PRINTCS( $\lceil n/2 \rceil$ )
        PRINTCS( $\lceil n/2 \rceil$ )
        for  $i \leftarrow 1$  to 20 do print("C")
```

Part (b)

(iv) **Algorithm PRINTDS** ( $n$  : integer)

```
    if  $n < 2$ 
        print("D")
    else
        for  $j \leftarrow 1$  to 4 do PRINTDS( $\lfloor n/2 \rfloor$ )
        for  $i \leftarrow 1$  to  $10n^k$  do print("D"),
```

where  $k$  is a nonnegative integer.

**Problem 2:** Bill is buying his wife a bouquet of carnations, roses, tulips, daises, and lilies. The bouquet will have 28 flowers, with

- at most 6 carnations,
- between 3 and 7 roses,
- between 2 and 11 tulips,
- at least 4 daises, and
- at least 1 lily.

How many different combinations of flowers satisfy these requirements? You need to use the counting method for integer partitions and show your work.

**Problem 3:** We have three sets  $P$ ,  $Q$ ,  $R$  with the following properties:

- (a)  $|P| = 3|Q|$  and  $|R| = |P|$ ,
- (b)  $|P \cap Q| = 11$ ,  $|Q \cap R| = 14$ ,  $|P \cap R| = 2|Q|$ ,
- (c)  $7 \leq |P \cap Q \cap R| \leq 17$ ,
- (d)  $|P \cup Q \cup R| = 100$ .

Use the inclusion-exclusion principle to determine the number of elements in  $P$ . Show your work. (Hint: You may get an equation with two unknowns, but one of them has only a few possible values.)

**Submission.** To submit the homework, you need to upload the pdf file into ilearn and Gradescope.