CS111 ASSIGNMENT 3

Problem 1: We want to tile an $n \times 1$ strip with 1×1 tiles that are green (G), blue (B), and red (R), and 3×1 purple tiles. Green and blue tiles cannot be next to each other, and no three green or blue tiles in a row are allowed (no GB, BG, GGG, and BBB). Give a formula for the number of such tilings. Your solution must include a recurrence equation (with initial conditions!), and a full justification. You do not need to solve it.

Problem 2: Solve the following recurrence equations:

a)

$$\begin{aligned} f_n &= f_{n-1} + 4f_{n-2} + 2f_{n-3} \\ f_0 &= 0 \\ f_1 &= 1 \\ f_2 &= 2 \end{aligned}$$

Show your work (all steps: the characteristic polynomial and its roots, the general solution, using the initial conditions to compute the final solution.)

b)

$$\begin{aligned} f_n &= f_{n-1} + 4f_{n-2} + 2f_{n-3} + 2n \\ f_0 &= 0 \\ f_1 &= 1 \\ f_2 &= 2 \end{aligned}$$

Show your work (all steps: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.) You can use your work from (a).

c)

$$\begin{aligned} t_n &= 2t_{n-1} + t_{n-2} + 2^n \\ t_0 &= 0 \\ t_1 &= 2 \end{aligned}$$

Show your work (all steps: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.)

Problem 3: (a) Let T_n denote the number of moves needed to solve the Tower of Hanoi problem with n discs. (a) Set up (and justify) a recurrence relation for the sequence T_n and solve it. (b) Use mathematical induction to verify the formula obtained in (a).

Submission. To submit the homework, you need to upload the pdf file into ilearn and Gradescope.