CS/MATH111 ASSIGNMENT 1

Problem 1: Give the exact and asymptotic formula for the number f(n) of letters "A" printed by Algorithm PRINTAS below. Your solution must consist of the following steps:

- (a) First express f(n) using a summation notation \sum .
- (b) Next, give a closed-form formula for f(n).
- (c) Finally, give the asymptotic value of the number of A's (using the Θ -notation.)

Include justification for each step.

Note: If you need any summation formulas for this problem, you are allowed to look them up, and do not need to prove.

Algorithm PRINTAS (n : integer)for $i \leftarrow 1$ to 3n + 1 do for $j \leftarrow 1$ to $(i + 2)^2$ do print("A") for $i \leftarrow 1$ to $(3n + 1)^2$ do for $j \leftarrow 1$ to i + 2 do print("A")

Problem 2: (a) Use mathematical induction to prove that $3^n \ge n2^n$ for $n \ge 0$. (Note: dealing with the base case may require some thought.)

(b) Consider a sequence defined recursively as $D_0 = 2$, $D_1 = 5$, and $D_n = 3D_{n-1} + D_{n-2}$ for $n \ge 2$. Prove that $D_n = O(3.35^n)$ and $D_n = \Omega(3.25^n)$.

Hint: First, prove by induction that $3.25^n \le D_n \le 2(3.35)^n$ for all $n \ge 0$.

Problem 3: Give the asymptotic values of the following functions, using the Θ -notation:

(a)
$$(n^2 + 5n - 10)(3n^3 - n)$$

(b)
$$3 + 2\sqrt{n} + 1/(\log^2 n)$$

- (c) $7n^3 \log n + 9n \log^5 n + 15n^3$
- (d) $15n^6 + n^2 \cdot 2^n + n^3 \log n$

(e)
$$n^3 \cdot \frac{7^n}{6} + n^5 \cdot 4^n$$

Justify your answers.

Submission. To submit the homework, you need to upload the pdf file into ilearn and Gradescope.

Reminders. Remember that only LATEX papers are accepted.