Math 142-2, Project 1

Both partners' names here

Introduction

This project looks at the equation for the undamped nonlinear pendulum

$$m\theta'' + k\sin\theta = 0 \qquad \theta(0) = \theta_0 \qquad \theta'(0) = 0$$

using a mixture of analytic and numerical techniques.

Problem 1

Show that this problem can be reduced to the simpler ODE

$$\theta'' + \sin\theta = 0 \qquad \theta(0) = \theta_0 \qquad \theta'(0) = 0$$

in the sense that the solution to the original problem can be readily deduced from the solution to this simpler problem.

Your solution goes here

Programming

To complete the remainder of this project, you will need to use a computer to numerically approximate the solution to the simplified ODE $\theta'' + \sin\theta = 0$, $\theta(0) = \theta_0$, $\theta'(0) = 0$. Use fourth order Runge-Kutta to approximate the ODE. The fourth order Runge-Kutta method for approximating the solution to an ODE $z' = f(z, t)$ is given by

$$\begin{aligned}
z_0 &= \langle x(0), x'(0)\rangle \\
z_{n+1} &= z_n + \frac{h}{6}(a + 2b + 2c + d) \\
a &= f(t_n, z_n) \\
b &= f\left(t_n + \frac{h}{2}, z_n + \frac{h}{2}a\right) \\
c &= f\left(t_n + \frac{h}{2}, z_n + \frac{h}{2}b\right) \\
d &= f(t_n + h, z_n + hc)
\end{aligned}$$

You will also need to compute an estimate of the period of your solution. A good way to do this is to compute the time when the sign of the solution first changes. An accurate estimate is obtained by connecting the last positive value and first negative value with a straight line and then choosing the time when this line crosses zero. You can use this information to estimate the period fairly accurately. **Please print out and submit your code with your solutions.**

Problem 2

For each $\theta_0 \in \{0.2, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.1, 3.14\}$, produce one plot showing (a) the solution x_{sol} to the nonlinear ODE, (b) the solution $x_{simp} = \theta_0 \cos t$ to the simplified linear ODE, and (c) an *adjusted* solution $x_{adj} = \theta_0 \cos \beta t$, where β is chosen so that the period matches the solution to the original ODE. (Note that the period is $2\pi/\beta$.) Observe that the period begins to deviate well before the shape does.

Your solution goes here

Problem 3

Produce a plot showing the error E_p in the period of the nonlinear pendulum and the error E_a in the adjusted approximation, where

$$E_p = \frac{1}{\beta} - 1$$

$$E_a = \frac{1}{\theta_0} \max_t |x_{sol}(t) - x_{adj}(t)|$$

Your solution goes here

Problem 4

At (approximately) what initial angle θ_0 does the period error E_p reach 1%? 10%?

Your solution goes here

Problem 5

At (approximately) what initial angle θ_0 does the adjusted error E_a reach 1%? 10%?

Your solution goes here

Problem 6

Why does the period get longer when the initial displacement is increased? (An intuitive explanation is sufficient.)

Your solution goes here

Problem 7

Provide an explanation for why a pendulum spends far more time with $|\theta|$ near its maximum than near zero, when the initial displacement is close to π. (An intuitive explanation is sufficient.)

Your solution goes here