Math 142-1, Midterm

Name: ______________________________ ID: ______________

Problem 1

Many physical formulas (e.g., $E = mc^2$, $E = \frac{hc}{\lambda}$, $V_T = \frac{kT}{q}$) contain fundamental constants (e.g., speed of light c, Plank's constant h, and Boltzmann's constant k), which is often seen as somewhat inconvenient. It was observed that since the choice of units is arbitrary, one could choose "more convenient" units. For example, if years are chosen for units of time and light-years (the distance light travels in a year) as the units for length, then $c = 1$. In fact, one can choose units so that $c = 1$, $h = 1$, and $k = 1$. The formulas above then "simplify" to $E = m$, $E = \frac{1}{\lambda}$, $V_T = \frac{T}{q}$. Omitting these constants from the formula turns out to be okay because they can always be unambiguously re-inserted later. For example, E has units of energy ($kg\, m^2\, s^{-2}$) but m has units of mass (kg). This can be balanced out with two factors of c (units of velocity, $m\, s^{-1}$) to recover the original equation $E = mc^2$. For each of the following "simplified" equations, determine what the originial equation was. A table listing units for all quantities is at right.

name	units
c	$m\, s^{-1}$
h	$kg\, m^2\, s^{-1}$
k	$kg\, m^2\, s^{-2}\, K^{-1}$
E	$kg\, m^2\, s^{-2}$
I	$kg\, s^{-2}$
λ	m
m	kg
ν	s^{-1}
p	$kg\, m\, s^{-1}$
q	C
T	K
V_T	$kg\, m^2\, s^{-2}\, C^{-1}$

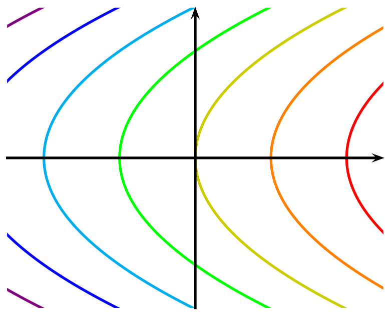
(a) $E^2 = m^2 + p^2$

(b) $E = \nu$

(c) $I = \dfrac{2\nu^3}{e^{\frac{\nu}{T}} - 1}$

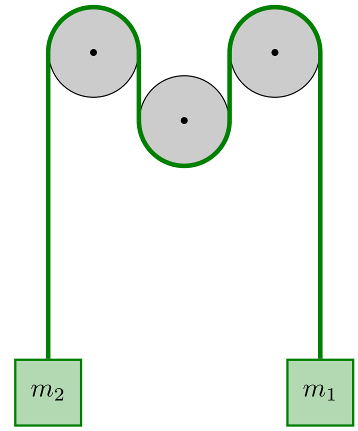
Problem 2

Several energy levels for a system are shown in the phase plane below. (a) Mark all stable ("•") and unstable ("○") equilibria. (b) Sketch energy contours corresponding to all unstable equilibria (energy contours may contain more than one component; be sure to sketch them all). (c) Add arrows to all contours, including the ones you have added. (d) Sketch the potential energy function and show the energy levels corresponding to the seven colored energy contours. (e) Construct a physical system that would exhibit motion consistent with the phase plane below.



Problem 3

A massless rope is wrapped around three identical pulleys of radius r as shown in the figure. A mass is attached to each end of the rope. The pulleys are located at $(-2r, 0)$, $(0, -\frac{3}{2}r)$, and $(2r, 0)$. When both masses are at the same height, they are located at $(-3r, -s)$ and $(3r, -s)$. The location of mass m_1 at any time is $(3r, x(t) - s)$.

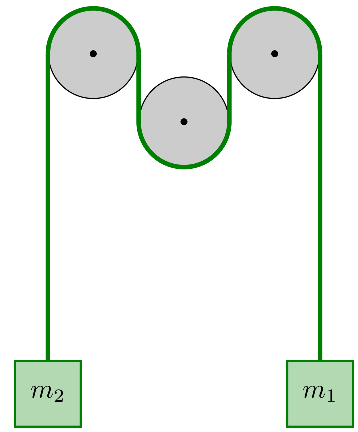


(a) What is the length ℓ of the rope?

(b) What is the total gravitational potential energy (ϕ)?

(c) The motion of the rope can be described by $\dot{x}(t)$. What is the total kinetic energy energy (KE)?

Problem 3 (continued)



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(d) What is the total energy (E)?

(e) Derive a second order ODE that describes the motion of the system.

(f) Find the equilibria of the system $(m_1 \neq m_2)$. Are they stable?

(g) Find the equilibria of the system $(m_1 = m_2)$. Are they stable?