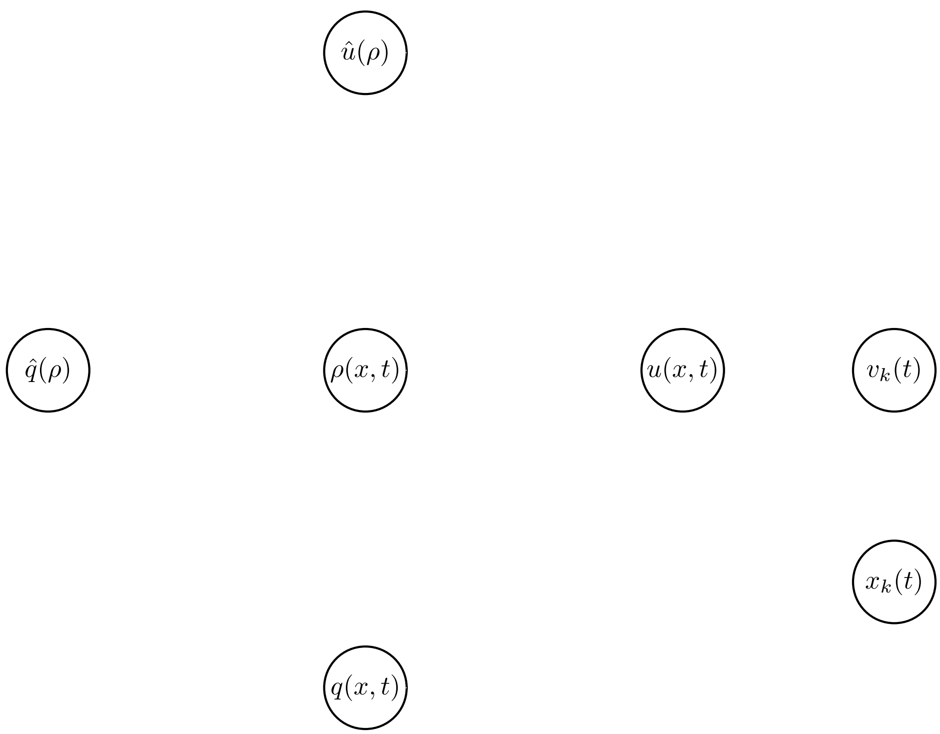
Math 142-2, Group work 7

Problem 1

For each row of the table, explain how you can obtain the wanted item from the quantities that are given to you. If you require initial condition or boundary conditions, state what you need. For example, for **(d)**, I can get $x_k(t)$ from $v_k(t)$ by solving the ODE $x_k'(t) = v_k(t)$ subject to the initial conditions $x_k(0) = x_{k0}$. It will help you to make a drawing with a circle for each quantity (you can use the circles below). Connect the circles with arrows as you find ways of computing them from each other. (You will build on this next week.)

#	Given	Want
(a)	$v_k(t)$	$u(x,t)$
(b)	$u(x,t)$	$v_k(t)$
(c)	$x_k(t)$	$v_k(t)$
(d)	$v_k(t)$	$x_k(t)$
(e)	$q(x,t)$, $u(x,t)$	$\rho(x,t)$
(f)	$\rho(x,t)$, $u(x,t)$	$q(x,t)$
(g)	$q(x,t)$, $\rho(x,t)$	$u(x,t)$
(h)	$\hat{q}(\rho)$, $\rho(x,t)$	$q(x,t)$
(i)	$\hat{u}(\rho)$, $\rho(x,t)$	$u(x,t)$



Problem 2

Suggest a traffic following model of the form $\dot{v}_k(t) = f(x_k, x_{k-1}, v_k, v_{k-1})$ that would lead to the relationship $\hat{u}(\rho) = u_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right)$.

Problem 3

Consider the model $\dot{v}_k(t) = -\lambda(x_k(t) - x_{k-1}(t) + d)$.

(a) What do λ and d represent?

(b) Why is $-\lambda$ more reasonable than λ?

(c) Show that no density-velocity relationship $\hat{u}(\rho)$ is implied by this model. (Hint: Show that $\hat{u}(\rho)$ implies $\dot{v}_k(t) = (v_k(t) - v_{k-1}(t))f(x_k(t) - x_{k-1}(t))$ for some function $f(x)$.)

(d) Do you think this model reasonably describes the behavior of drivers?

(e) Construct a traffic following model with a density-velocity relationship $\hat{u}(\rho)$ such that drivers try to maintain a fixed following time. (For example, drivers always try to stay two seconds behind the car in front.) Is this always possible?

(f) Is the model $\dot{v}_k(t) = \frac{v_k(t) - v_{k-1}(t)}{x_k(t) - x_{k-1}(t) + d}$ plausible?