Math 142-2, Group work 6

Problem 1

Each of the derivations below is incorrect. Find the mistakes.

(a) Problem: Let $f(x,y) = 2x^2 + y^3$ and $g(x,y) = f(x,y) + f(y,x)$. Compute $g_x(1,2)$.
Solution 1:

$$\begin{aligned} f(x,y) &= 2x^2 + y^3 \\ f_x(x,y) &= 4x \\ f_x(1,2) &= 4 \\ f_x(2,1) &= 8 \\ g(x,y) &= f(x,y) + f(y,x) \\ g_x(x,y) &= f_x(x,y) + f_x(y,x) \\ g_x(1,2) &= f_x(1,2) + f_x(2,1) \\ &= 4 + 8 = 12 \end{aligned}$$

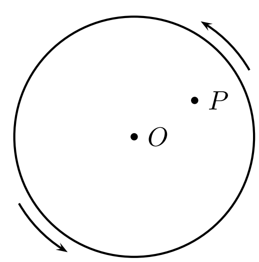
Solution 2:

$$\begin{aligned} g(x,y) &= f(x,y) + f(y,x) \\ &= (2x^2 + y^3) + (2y^2 + x^3) \\ &= 2x^2 + x^3 + 2y^2 + y^3 \\ g_x(x,y) &= 4x + 3x^2 \\ g_x(1,2) &= 4 + 3 = 7 \end{aligned}$$

(b) Let $K = \frac{1}{2}mv^2$ be the kinetic energy of a particle. Define the quantity $A = \frac{\partial^2 K}{\partial m^2}$. In some contexts, velocity is an inconvenient variable, and momentum $p = mv$ is preferred instead. In such cases, one would write $K = \frac{p^2}{2m}$. Compute A if $m = 2$ and $p = 4$.
Solution 1: From the momentum form of K, $A = \frac{\partial^2 K}{\partial m^2} = \frac{p^2}{m^3} = \frac{16}{8} = 2$.
Solution 2: From the velocity form of K, it is clear that $A = 0$.

(c) Let $K = \frac{1}{2}m\dot{x}^2$ be the kinetic energy of a particle that is falling from rest under gravity ($\ddot{x} = -g$). Does K depend on time?
Solution 1: No; $\frac{\partial K}{\partial t} = 0$, so K does not depend on time.
Solution 2: Yes; $\frac{dK}{dt} = m\dot{x}\ddot{x} \neq 0$, so K does depend on time.

Problem 2



A disk spins about the origin, as shown in the diagram.

(a) Let P be a dot painted on the disk, initially at location (x_0, y_0). Assuming the angular velocity $\dot{\theta}$ is constant and that the disk makes one complete revolution after time T, find the velocity and position of P at an arbitrary time t.

(b) Under the assumptions of part **(a)**, find the velocity field $\vec{u}(\vec{x}, t)$.

(c) Does the velocity change with time?

(d) Assume instead that the angular velocity decays exponentially. That is, $\ddot{\theta} = -r\dot{\theta}$. The disk completes its first rotation at time T, by which time its angular velocity has halved. What is r? Find the velocity and position of P at an arbitrary time t.

(e) Under the assumptions of part **(c)**, find the velocity field $\vec{u}(\vec{x}, t)$.