Math 142-2, Group work 1

Problem 1

This group work activity is based around exploring the second order ordinary differential equation

$$tx'' + x' + tx = \frac{1}{4}\sin t.$$

Write this second order ODE as a first order ODE of the form $z' = f(z,t)$ by introducing a new variable $v = x'$. Note that z in our case will need to be the vector $\langle x, v\rangle$. Find $f(z,t)$.

Problem 2

Write a Matlab function which takes two arguments and computes $f(z,t)$ (note that the argument z and the result will be vectors). You are not required to use Matlab; you may use another language if you prefer. You will need to be able to generate line plots.

Problem 3

Plot the vector valued function $f(\langle\cos t, -\sin t\rangle, t)$ for $t \in [0, 4\pi]$ using the Matlab function you wrote. Plot the function at around hundred or so sample points. (Note that you will have difficulty at $t = 0$; one simple solution is to plot it at a tiny value like $t = 10^{-10}$ instead.)

Problem 4

When solving ODE's numerically, we compute approximate values for $z_n \approx z(t_n)$ at a large number N of sample times $t_0 \dots t_N$ in the interval $[0, T]$. The sample points are equally spaced ($t_{n+1} = t_n + h$ for fixed h), with $t_0 = 0$ and $t_N = T$. The forward Euler method for approximating the solution to an ODE $z' = f(z,t)$ is given by

$$z_0 = \langle x(0), x'(0)\rangle$$
$$z_{n+1} = z_n + hf(t_n, z_n)$$

Compute $z_0 \dots z_N$ for $N = 400$, $T = 20$, $x(0) = 1$. (What is $x'(0)$?)

Problem 5

Generate a plot of the points $(t_n, x(t_n))$.

Problem 6

Approximate the maximum and minimum values of $x(t)$ over the interval $[0, T]$.