# Math 142-2, Homework 1

Your name here

## Problem 5.8

**(a)** Show that $x = c_1 \cos \omega t + c_2 \sin \omega t$ is the general solution of $mx'' = -kx$. What is the value of $\omega$?

**Your solution goes here**

**(b)** Show that an equivalent expression for the general solution is $x = B\cos(\omega t + \theta_0)$. How do $B$ and $\theta_0$ depend on $c_1$ and $c_2$?

**Your solution goes here**

## Problem 6.2

Suppose a quantity $y$ having dimensions of time is *only* a function of $k$ and $m$. Do not use constants with units.

**(a)** Give an example of a possible dependence of $y$ on $k$ and $m$.

**Your solution goes here**

**(b)** Can you describe the most general dependence that $y$ can have on $k$ and $m$?

**Your solution goes here**

## Problem 7.2

A weight (of unknown mass) is placed on a vertical spring (of unknown spring constant) compressing it by $2.5\,cm$. What is the natural frequency of oscillation of this spring-mass system?

**Your solution goes here**

## Problem 9.3

Suppose that a mass $m$ were attached between two walls a distance $d$ apart (see figures in book).

**(a)** Briefly explain why it is *not* necessary for $d = \ell_1 + \ell_2$.

**Your solution goes here**

**(b)** What position of the mass would be called the equilibrium position of the mass? If both springs are identical, where should the equilibrium position be? Show that your formula is in agreement.

**Your solution goes here**

**(c)** Show that the mass executes simple harmonic motion about its equilibrium position.

**Your solution goes here**

**(d)** What is the period of the oscillation?

**Your solution goes here**

**(e)** How does the period of oscillation depend on $d$?

**Your solution goes here**

# Problem 9.4

Suppose that a mass $m$ were attached to two springs in *parallel* (see figure in book).

**(a)** What position of the mass would be called the equilibrium position of the mass?

**Your solution goes here**

**(b)** Show that the mass executes simple harmonic motion about its equilibrium position.

**Your solution goes here**

**(c)** What is the period of the oscillation?

**Your solution goes here**

**(d)** If the two springs were to be replaced by one spring, what would be the unstretched length and spring constant of the new spring such that the motion would be *equivalent*?

**Your solution goes here**

# Problem 9.5

Suppose that a mass $m$ were attached to two springs in *series* (see figure in book).

**(a)** What position of the mass would be called the equilibrium position of the mass?

**Your solution goes here**

**(b)** Show that the mass executes simple harmonic motion about its equilibrium position.

**Your solution goes here**

**(c)** What is the period of the oscillation?

**Your solution goes here**

**(d)** If the two springs were to be replaced by one spring, what would be the unstretched length and spring constant of the new spring such that the motion would be *equivalent*?

**Your solution goes here**

# Problem 9.6

Consider two masses each of mass $m$ attached between two walls a distance $d$ apart (see figure in book). Assume that all three springs have the same spring constant and unstretched length.

**(a)** Suppose that the left mass is a distance $x$ from the left wall and the right mass a distance $y$ from the right wall. What position of each mass would be called the equilibrium position of the system of masses?

**Your solution goes here**

**(b)** Show that the distance between the masses oscillates. What is the period of the that oscillation?

**Your solution goes here**

**(c)** Show that $x - y$ executes simple harmonic motion with a period of oscillation different from b.

**Your solution goes here**

**(d)** If the distance between the two masses remained constant, describe the motion that could take place both qualitatively and quantitatively.

**Your solution goes here**

**(e)** If $x = y$, describe the motion that could take place both qualitatively and quantitatively.

**Your solution goes here**

# Additional Problem 1

Maxwell's equations are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

A particle with charge $q$ and velocity $\mathbf{v}$ in this field will experience a Lorentz force

$$\mathbf{f} = q(\mathbf{E} + (\mathbf{v} \times \mathbf{B}))$$

Additionally, the total charge $Q$ in some volume of space $\Omega$ is

$$Q = \int_\Omega \rho \, dV$$

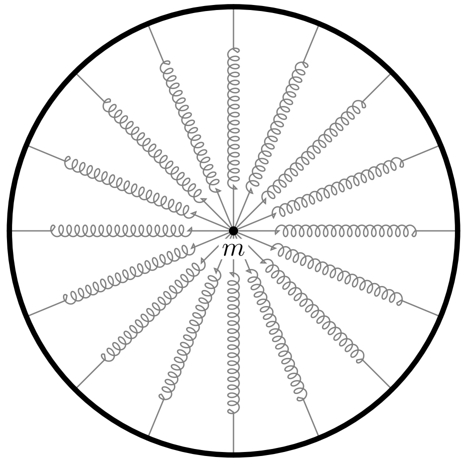
If $q = [C]$ and $Q = [C]$ both have units of charge (the unit $C$ is the Coulomb), deduce the units of the quantities in the table below. You may assume units for $\mathbf{f}$ and $\mathbf{v}$.

| var | meaning |
|---|---|
| $\mathbf{E}$ | electric field |
| $\mathbf{B}$ | magnetic field |
| $\rho$ | charge density |
| $\mathbf{J}$ | current density |
| $\varepsilon_0$ | permittivity of free space |
| $\mu_0$ | permeability of free space |
| $Q$ | total charge |
| $q$ | particle charge |
| $\mathbf{v}$ | particle velocity |
| $\mathbf{f}$ | force on particle |

**Your solution goes here**

# Additional Problem 2

A particle of mass $m$ is connected to a circle of radius $r$ by a very large number of identical and equally spaced springs. The particle is free to move around in three dimensions, but the circle is fixed.




**(a)** The springs have zero rest length but are free to rotate at both ends. Formulate a force law for one of the springs. That is, express $\mathbf{f}$ in terms of the mass position $\mathbf{x}$, assuming the other end of the spring is fixed at $\mathbf{z}$. (If your force does not work with the calculations below, choose a new one.)

**Your solution goes here**

**(b)** A force is *conservative* if it comes from an energy. That is, there exists a scalar function $\phi(\mathbf{x})$ such that $\mathbf{f} = -\nabla\phi$. Determine $\phi$ for your force.

**Your solution goes here**

**(c)** What is the total potential energy if there are $n$ equally spaced identical springs, as $n \to \infty$? Remember that the mass moves in three dimensions.

**Your solution goes here**

**(d)** What is the corresponding force?

**Your solution goes here**

**(e)** Solve for the path of the mass, if initially at location $\mathbf{x}_0$ with velocity $\mathbf{v}_0$.

**Your solution goes here**