Math 142-1, Project 1

Both partners' names here

Introduction

This project looks at the equation for the undamped nonlinear pendulum

$$m\theta'' + k\sin\theta = 0 \qquad \theta(0) = \theta_0 \qquad \theta'(0) = 0$$

using a mixture of analytic and numerical techniques.

Problem 1

Show that this problem can be reduced to the simpler ODE

$$\theta'' + \sin\theta = 0 \qquad \theta(0) = \theta_0 \qquad \theta'(0) = 0$$

in the sense that the solution to the original problem can be readily deduced from the solution to this simpler problem.

Your solution goes here

Programming

To complete the remainder of this project, you will need to use a computer to numerically approximate the solution to the simplified ODE $\theta'' + \sin\theta = 0$, $\theta(0) = \theta_0$, $\theta'(0) = 0$. You may implement your own routines to do this, or you may use an existing solver (such as the ODE solvers in MATLAB). If you choose to implement your own, I would recommend using fourth order Runge-Kutta. You may also find it useful to compute an estimate of the period of your solution, since this will be needed frequently and will be tedious to estimate by hand. **Please print out and submit your code with your solutions.**

Problem 2

For each $\theta_0 \in \{0.2, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.1, 3.14\}$, produce one plot showing (a) the solution x_{sol} to the nonlinear ODE, (b) the solution $x_{simp} = \theta_0 \cos t$ to the simplified linear ODE, and (c) an *adjusted* solution $x_{adj} = \theta_0 \cos\beta t$, where β is chosen so that the period matches the solution to the original ODE. (Note that the period is $2\pi/\beta$.) Observe that the period begins to deviate well before the shape does.

Your solution goes here

Problem 3

Produce a plot showing the error E_p in the period of the nonlinear pendulum and the error E_a in the adjusted approximation, where

$$E_p = \frac{1}{\beta} - 1$$
$$E_a = \frac{1}{\theta_0} \max_t |x_{sol}(t) - x_{adj}(t)|$$

Your solution goes here

Problem 4

At (approximately) what initial angle θ_0 does the period error E_p reach 1%? 10%?

Your solution goes here

Problem 5

At (approximately) what initial angle θ_0 does the adjusted error E_a reach 1%? 10%?

Your solution goes here

Problem 6

Why does the period get longer when the initial displacement is increased?

Your solution goes here

Problem 7

Provide an explanation for why a pendulum spends far more time with $|\theta|$ near its maximum than near zero, when the initial displacement is close to π.

Your solution goes here