Problem 1

A massless and frictionless pulley of radius r centered at the origin has a rope of length ℓ draped over it. The position of the rope can be described by x(t), the distance of the right end of the rope below the center of the pulley. The rope has total mass m, uniformly distributed along its length.

(a) What are the coordinates of the endpoints of the rope?

(b) What is the total gravitational potential energy (ϕ) of the rope?

(c) The motion of the rope can be described by $\dot{x}(t)$. What is the total kinetic energy energy (KE) of the rope?

(d) What is the total energy of the rope?

(e) Derive a second order ODE that describes the motion of the system.

(f) Find the equilibria of the system. Are they stable?