Math 142-1, Group work 10

Problem 1

This problem investigates *heat capacity*, the amount of energy that is required to increase the temperature of a substance by a specified amount. We have seen before that an expanding gas will cool down by doing $P\Delta V$ work on the walls of the container. Thus, changing the volume of the gas will alter both its internal energy U, kinetic energy KE, and temperature T. To avoid this problem, we will hold the volume of the gas constant. Imagine that we are heating a closed container of gas.

Molar heat capacity ($C_{V,m}$) is the amount of energy that must be applied to a fixed quantity (as measured by the number of particles, or moles) of gas in order to increase its temperature by a fixed amount. By our assumption, it is a constant *for each type of gas*. Here, I want to investigate how molar heat capacity depends on the gas. That is, $C_{V,m} = \frac{\Delta U}{n\Delta T}$.

(a) In class we assumed that $KE = \alpha U$. Express the heat molar heat capacity in terms of this fraction α. (The heat capacity of an ideal gas does not depend on things like T, P, or V.)

(b) How is the molar heat capacity affected if I replace the gas molecules with ones that are twice as massive but are otherwise identical? (For example, replacing neon ($20\,g/mol$) with argon ($40\,g/mol$)).

(c) The heat capacity of a molecular gas normally depends on the temperature. At very low temperature, there is not enough energy to cause molecules to vibrate or rotate. At higher temperatures, the molecules might start to rotate but not vibrate. At still higher temperatures, the molecules may rotate and vibrate. (The fact this rotation and vibration do not occur at low energy is not what we would expect. This is a quantum mechanical affect.) With this in mind, how would you expect molar heat capacity to change with temperature?

Problem 2

An idea gas is contained within a box with a moveable wall. This wall is able to move slowly in or out, which decreases or increases the gas volume. Assume the walls are well-insulated and the wall moves very slowly. Assume that a constant fraction α of the total energy of the gas is translational energy. (If additional translational energy is added, some of that will be converted into another form, such as by rotating or vibrating the molecules, so that the fraction α stays the same.)

(a) Show that $PV^\gamma = \text{const}$, where $\gamma = 1 + \frac{2}{3}\alpha$.

(b) How is T related to V under these circumstances?

Problem 3

In class, we assumed that gravity did not matter. In this problem, we will explore a case where it does matter. We will model the atmosphere and try to predict the pressure and density profiles. Assume uniform gravity acceleration g (that is, assume the atmosphere is thin relative to the radius of the earth). We can neglect the curvature of the earth, thinking instead of the surface being an infinite plane with uniform gravity holding the gas to it. We will also assume for simplicity that the gas is all of one type, and that type is monotonic (e.g., argon).

(a) Consider a single particle of gas. This particle has mass m has velocity v_0 when it bounces off the ground. How high will the particle go before falling back to the surface of the earth?

(b) How long does it take to return to the earth?

(c) How much momentum is transferred to the earth by the particle each time it bounces?

(d) Lets fix an area of the earth's surface A. Assume there are N molecules of air above this area of the surface and that this stays the same over time. What will be the pressure over the surface?