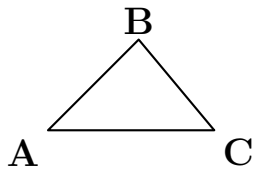


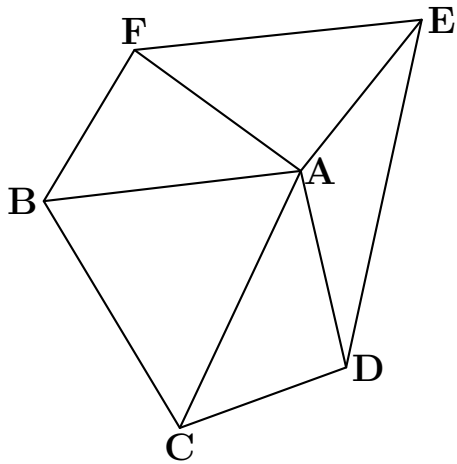
Triangle Meshes and Data Structures

1 Triangle representation



A basic triangle is defined by three vertices $A = (A_x, A_y, A_z)$, $B = (B_x, B_y, B_z)$, and $C = (C_x, C_y, C_z)$. This requires 9 floating-point numbers or about 36 bytes.

1.1 Triangle mesh



One way to represent this is with a list of triangles:

$$(A_x A_y A_z B_x B_y B_z C_x C_y C_z), (A_x A_y A_z C_x C_y C_z D_x D_y D_z), \dots$$

If there are m vertices and n triangles, then this representation requires storing $9n$ numbers. Instead, we could store vertices in one array:

$$(A_x A_y A_z)(B_x B_y B_z)(C_x C_y C_z)(D_x D_y D_z) \dots$$

Then, we can represent a vertex by its integer index into that array. The triangles above are then

$$(012)(023)(034)(045)(051)$$

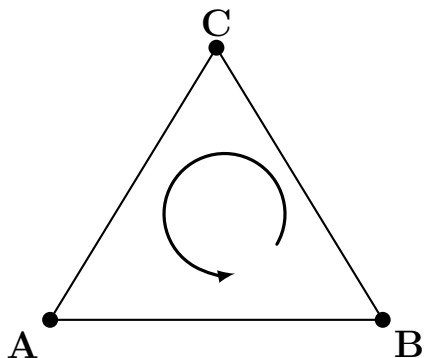
This representation requires $3m + 3n$ numbers. In code, this might look like

```
std::vector<vec3> vertices;  
std::vector<ivec3> triangles;
```

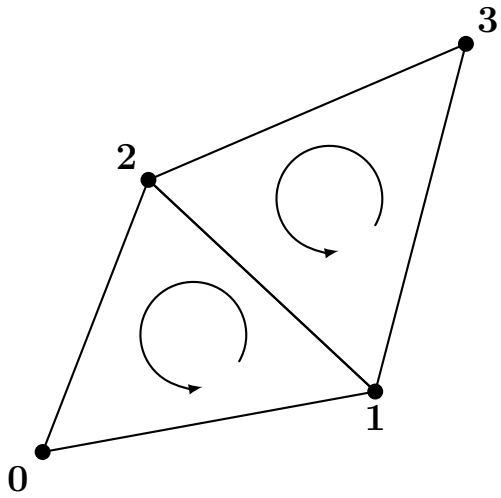
To see how m and n compare in practice, consider an $s \times s$ grid of squares, each broken into two triangles. Then, $m = (s + 1)^2$ and $n = 2s^2$, so that $n/m \rightarrow 2$. The size is thus $3m + 3n \approx 4.5n$, which is about half what the original representation required.

2 Normals

When computing triangle normals, we need to know which side of the face is “outside.” This requires us to have an *orientation* for the triangle. The standard convention is that if we are looking at the triangle from the outside, then the vertices will be listed in counterclockwise order. With this convention, $\vec{N} = (B - A) \times (C - A)$ points outward along the normal direction. Normalizing it gives the normal direction \vec{n} .



3 Orientation consistency



Our triangle normals should be consistent, in the sense that neighboring triangles agree on which side is outside. This can be tested by taking the dot product of the triangle normals, but this fails along sharp edges. Instead, we can actually determine this directly from the triangle indices. Observe that the edge 1, 2 is traced in opposite orders by the two triangles.