Segments in homogeneous coordinates

1 Segments in 3D

When clipping a line segment between two points A and B in 3D space, we find an intermediate point P by interpolating

$$P = \lambda A + (1 - \lambda) B$$

where $0 \leq \lambda \leq 1$. When dealing with homogeneous coordinates $\overline{A}$ and $\overline{B}$, we interpolated using

$$\overline{P} = \gamma \overline{A} + (1 - \gamma) \overline{B}.$$

Converting from homogeneous coordinates back to 3D coordinates involves division, so it is sot at all obvious that this interpolation actually traces a line. Interpolating in these ways gives us two different interpolation weights λ and γ, and we need to decide which to use. We begin by noting the relationship between the two coordinates.

$$\overline{A} = \begin{pmatrix} w_a A \\ w_a \end{pmatrix} \qquad \overline{B} = \begin{pmatrix} w_b B \\ w_b \end{pmatrix} \qquad \overline{P} = \begin{pmatrix} w_p P \\ w_p \end{pmatrix}$$

Then, P is obtained from $\overline{P}$ by dividing off w_p.

$$\begin{aligned} \overline{P} &= \gamma \overline{A} + (1 - \gamma) \overline{B} \\ \begin{pmatrix} w_p P \\ w_p \end{pmatrix} &= \begin{pmatrix} \gamma w_a A + (1 - \gamma) w_b B \\ \gamma w_a + (1 - \gamma) w_b \end{pmatrix} \\ P &= \frac{\gamma w_a A + (1 - \gamma) w_b B}{\gamma w_a + (1 - \gamma) w_b} \\ &= \underbrace{\frac{\gamma w_a}{\gamma w_a + (1 - \gamma) w_b}}_{\lambda} A + \underbrace{\frac{(1 - \gamma) w_b}{\gamma w_a + (1 - \gamma) w_b}}_{1-\lambda} B \\ \lambda &= \frac{\gamma w_a}{\gamma w_a + (1 - \gamma) w_b} \end{aligned}$$

This is just interpolation in 3D coordinates. Further, we see the relationship between the two weights. It turns out that λ is the image space barycentric coordinate, and γ is the world space barycentric coordinate. You will need both. In particular, image space interpolation in OpenGL corresponds to “noperspective,” and world space interpolation corresponds to “smooth.”