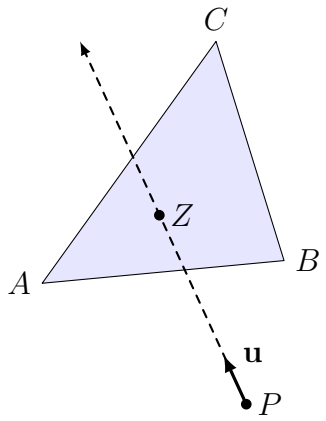
Ray-triangle intersection

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1 Setting up the problem



We wish to intersect triangle ABC with a ray having endpoint P and direction $\mathbf{u}$. Let Z be this intersection point. Since Z is on the ray, we have

$$Z = P + t\mathbf{u} \qquad t \geq 0.$$

Since Z is also in the triangle, we can write

$$Z = \alpha A + \beta B + \gamma C,$$

where α, β, γ are barycentric coordinates and $\alpha + \beta + \gamma = 1$. Eliminating α and Z we get

$$\begin{aligned} P + t\mathbf{u} &= A + \beta(B - A) + \gamma(C - A) \\ \mathbf{y} + t\mathbf{u} &= \beta\mathbf{v} + \gamma\mathbf{w} \end{aligned}$$

where we have defined $yy = P - A$, $\mathbf{v} = B - A$, and $\mathbf{w} = C - A$ for convenience. This leaves us with three equations (we have a vector equation with three components) and three unknowns (t, β, γ). We could solve this by setting up a 3×3 matrix-vector equation, but we will instead use a trick.

2 Solving the problem

Let $\mathbf{n} = \mathbf{u} \times \mathbf{v}$ so that $\mathbf{n} \cdot \mathbf{u} = \mathbf{n} \cdot \mathbf{v} = 0$. Then,

$$\begin{aligned} \mathbf{y} &= \beta\mathbf{v} + \gamma\mathbf{w} - t\mathbf{u} \\ \mathbf{y} \cdot \mathbf{n} &= \beta(\mathbf{v} \cdot \mathbf{n}) + \gamma(\mathbf{w} \cdot \mathbf{n}) - t(\mathbf{u} \cdot \mathbf{n}) \\ \mathbf{y} \cdot \mathbf{n} &= \gamma(\mathbf{w} \cdot \mathbf{n}) \\ \gamma &= \frac{\mathbf{y} \cdot \mathbf{n}}{\mathbf{w} \cdot \mathbf{n}} \end{aligned}$$

The solution for β and t can be obtained similarly. Finally, we get $\alpha = 1 - \beta - \gamma$. We have now found Z, but we have not actually determined whether there was an intersection.

The point Z is on the ray if $t \geq 0$. (Otherwise, the triangle is behind the ray.) The point Z is in the triangle if $\alpha, \beta, \gamma \geq 0$.