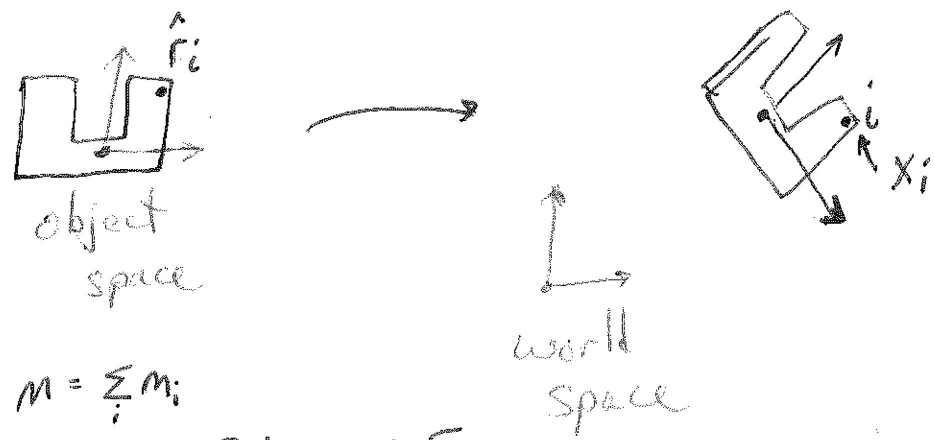
x



$$M = \sum_i m_i$$

$$x_i = x + R\hat{r}_i = x + r_i$$

$$\sum_i m_i \hat{r}_i = \sum_i m_i r_i = 0$$

$$v_i = v + \omega \times r_i$$

$$\dot{R} = \omega^* R$$

$$\dot{x} = v$$

$$P = \sum_i m_i v_i = \sum_i m_i (v + \omega \times r_i)$$

$$= \left(\sum_i m_i\right) v + \sum_i m_i \, \omega \times r_i$$

$$= Mv + \omega \times \underbrace{\sum_i m_i r_i}_{0}$$

$$= Mv$$

$$L = \sum_i r_i \times m_i v_i = \sum_i r_i \times m_i (V + \omega \times r_i)$$

$$= \sum_i r_i \times m_i V + \sum_i r_i \times m_i (\omega \times r_i)$$

$$= \underbrace{\left(\sum_i m_i r_i\right)}_{0} \times V + \sum_i r_i \times m_i (\omega \times r_i)$$

$$\underbrace{u^*}_{\text{matrix}} V = u \times V$$

$$= \underbrace{\left(\sum_i m_i r_i^* r_i^{*T}\right)}_{I} \omega = I\omega$$

inertia tensor

$$I = \sum_i m_i r_i^* r_i^{*T}$$

$$\omega \times r_i$$
$$= - r_i \times \omega$$
$$= - r_i^* \omega$$
$$= r_i^{*T} \omega$$

δ = identity

$$L = I\omega$$

$$r = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad r^* r^{*T} = \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix} \begin{pmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{pmatrix}$$

$$= \begin{pmatrix} z^2 + y^2 & -xy & -xz \\ -xy & z^2 + x^2 & -yz \\ -xz & -yz & y^2 + x^2 \end{pmatrix}$$

$$= \begin{pmatrix} x^2+y^2+z^2 & & \\ & x^2+y^2+z^2 & \\ & & x^2+y^2+z^2 \end{pmatrix} - \begin{pmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{pmatrix}$$

$$= \delta r^T r - r r^T$$

$$r^* r^{*T} = \delta r^T r - r r^T$$

$$(R\hat{r})^* (R\hat{r})^{*T} = \delta (R\hat{r})^T (R\hat{r}) - (R\hat{r})(R\hat{r})^T$$

$$= \delta \hat{r}^T \underbrace{R^T R}_{\delta} \hat{r} - R \hat{r}\hat{r}^T R^T$$

$\underline{r_i = R\hat{r}_i}$

$$= \delta \underline{\hat{r}^T \hat{r}} - R \hat{r}\hat{r}^T R^T$$

$$= R\left(\delta \hat{r}^T r - \hat{r}\hat{r}^T\right) R^T = R\, \hat{r}^* \hat{r}^{*T} R^T$$

$$I = \sum_i m_i r_i^* r_i^{*T} = \sum_i m_i (R\hat{r}_i)^* (R\hat{r}_i)^{*T}$$

$$= \sum_i m_i R \left(\hat{r}_i^* r_i^{*T}\right) R^T$$

$$= R\left(\sum_i m_i \hat{r}_i^* \hat{r}_i^{*T}\right) R^T \qquad \hat{I} = \sum_i m_i \hat{r}_i^* \hat{r}_i^{*T}$$

$$= R \hat{I} R^T$$

$$\boxed{\begin{aligned} f &= \dot{p} = m\dot{v} \\ \tau &= \dot{L} \end{aligned}} \qquad \boxed{\begin{aligned} I &= R\hat{I}R^T \\ L &= I\omega \end{aligned}} \qquad \boxed{\begin{aligned} \dot{x} &= v \\ \dot{R} &= \omega^* R \end{aligned}}$$

↑
torque

$\dot{x} = v$ $\dot{R} = \omega^* R$ store: x^n v^n R^n $\underline{L^n}$

$$\frac{x^{n+1} - x^n}{\Delta t} = v^n \quad \frac{R^{n+1} - R^n}{\Delta t} = (\omega^n)^* R^n \quad \omega^n = (I^n)^{-1} L^n \quad I^n = R^n \hat{I} (R^n)^T$$

$$\frac{L^{n+1} - L^n}{\Delta t} = \tau^n \quad \frac{v^{n+1} - v^n}{\Delta t} = \frac{f^n}{m}$$