## CS 230, Quiz 4

## Solutions

You will have $\mathbf{8}$ minutes to complete this quiz. No books, notes, or other aids are permitted. There is another problem on the back.

## Problem 1 (3 points)

A square with a letter (shown in the diagram labeled "orig" below) is transformed into each of the configurations (a)-(c). In each case, identify the type of transform and, if possible, find a $3 \times 3$ homogeneous transform matrix corresponding to it. In each case, identify the transform as a $\mathrm{R}=$ rotation, $\mathrm{T}=$ translation, $\mathrm{S}=$ uniform scale, $\mathrm{X}=$ none of these. $\mathrm{R}, \mathrm{S}$, and T can be combined. The most restrictive option should be chosen. Thus, a transform that can be accomplished by a combination of rotation and uniform scale should be described as $\mathrm{R}+\mathrm{S}$, not as X .
2




(a) R. $\left(\begin{array}{ccc}0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1\end{array}\right)$
(b) T. $\left(\begin{array}{ccc}1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1\end{array}\right)$
(c) $\mathrm{S}+\mathrm{T} \cdot\left(\begin{array}{ccc}2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1\end{array}\right)$

## Problem 2 (1 points)

What does the $4 \times 4$ homogeneous transform matrix $\mathbf{M}$ do?

$$
\mathbf{M}=\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 2
\end{array}\right)
$$

This problem is actually a little tricky, since we have not discussed the effects of putting something other than 1 in the bottom right. Nevertheless, we can simply examine how it changes a point $(x, y, z)$.

$$
\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 2
\end{array}\right)\left(\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right)=\left(\begin{array}{l}
x \\
y \\
z \\
2
\end{array}\right) \equiv\left(\begin{array}{c}
\frac{x}{2} \\
\frac{y}{2} \\
\frac{z}{2}
\end{array}\right)
$$

This matrix behaves as a uniform scale by $\frac{1}{2}$. Indeed, since scaling a homogeneous point makes no difference (since we will eventually divide off the $w$ ), scaling a transformation also makes no difference.

$$
\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 2
\end{array}\right) \equiv\left(\begin{array}{cccc}
\frac{1}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

This is the form we would expect from a scale matrix.

