Line Rasterization

University of California Riverside
- Object oriented
  - for each object...

- Image oriented
  - for each pixel...
What is rasterization?

Rasterization is the process of determining which pixels are “covered” by the primitive
Rasterization

- **In**: 2D primitives (floating point)
- **Out**: covered pixels (integer)
- **Must be fast** (called **many times**)
- **Visually pleasing**
  - lines have constant width
  - lines have no gaps
DDA algorithm for lines

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- Plot line $y = mx + b$
- For each $x$:
  - $y = mx + b$
  - turn on pixel $(x, \text{round}(y))$
DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
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  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$
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- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$
  - $y_{i+1} = mx_{i+1} + b$
  - $= m(x_i + 1) + b$
  - $= y_i + m$
DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$
  
  $$y_{i+1} = mx_{i+1} + b$$
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- Each time:
DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$
    
    $$y_{i+1} = mx_{i+1} + b$$
    
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- Each time:
  - Increment $x$
DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$
  
  $$y_{i+1} = mx_{i+1} + b$$
  
  $$= m(x_i + 1) + b$$
  
  $$= y_i + m$$

- Each time:
  - Increment $x$
  - Add $m$ to $y$
DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}, \ x_{i+1} = x_i + 1, \ x_n = \text{end}$

\[
y_{i+1} = mx_{i+1} + b \\
= m(x_i + 1) + b \\
= y_i + m
\]

- Each time:
  - Increment $x$
  - Add $m$ to $y$
  - turn on pixel $(x_i, \text{round}(y_i))$
DDA algorithm for lines

What if $|m| > 1$?

Loop over $y$, compute and round $x$. Increment $y$ by $m$.
DDA algorithm for lines

- What if $|m| > 1$?
- Increment $y$ by $m$
DDA algorithm for lines

- What if $|m| > 1$?
- Increment $y$ by $m$
- $\text{round}(y)$ may skip an integer
  - gap in the line
What if $|m| > 1$?
- Increment $y$ by $m$
- $\text{round}(y)$ may skip an integer
  - gap in the line
- Swap the roles of $x$ and $y$
  - Loop over $y$, compute and round $x$
DDA algorithm for lines - limitations

- Must round for each pixel
- very slow
- Only use ops: $+, -, \times$
  - Even better: $+, -$
Rasterization choices

- Thin, no gaps
- Still have choices
Midpoint algorithm

- Assume $0 \leq m \leq 1$
- Move from left to right
- Choose between $(x + 1, y)$ and $(x + 1, y + 1)$

$$y = y_0$$

for $x = x_0, \ldots, x_1$ do

    draw($x, y$)
    if ⟨condition⟩ then
        $y \leftarrow y + 1$
Check midpoint location
Check midpoint location
Check midpoint location
Criterion

Implicit line equation:

\[ f(x) = n \cdot (x - x_0) = 0 \]
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\[ f(\mathbf{x}) = \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_0) = 0 \]

Evaluate \( f \) at midpoint:

\[ f \left( x + 1, y + \frac{1}{2} \right) < 0 \]
Implicit line equation:

\[ f(x) = n \cdot (x - x_0) = 0 \]

Evaluate \( f \) at midpoint:

\[ f \left( x + 1, y + \frac{1}{2} \right) < 0 \]
Midpoint algorithm \((0 \leq m \leq 1)\)

\[
y \leftarrow y_0 \\
\text{for } x = x_0, \ldots, x_1 \text{ do} \\
\quad \text{draw}(x, y) \\
\quad \text{if } f(x + 1, y + \frac{1}{2}) < 0 \text{ then} \\
\quad \quad y \leftarrow y + 1
\]
Efficiency: incremental update

- Compute initial $f(x, y)$
- Compute next by updating previous
- Update with *one* addition

$$f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + (x_0y_1 - x_1y_0)$$
Efficiency: incremental update

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\begin{align*}
  f(x, y) &= (y_0 - y_1)x + (x_1 - x_0)y + (x_0y_1 - x_1y_0) \\
  f(x + 1, y) &= f(x, y) + (y_0 - y_1)
\end{align*}
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f(x + 1, y) &= f(x, y) + (y_0 - y_1) \\
f(x + 1, y + 1) &= f(x, y) + (y_0 - y_1) + (x_1 - x_0)
\end{align*}
\]
Efficiency: incremental update

\[
y \leftarrow y_0 \\
d \leftarrow f(x_0 + 1, y_0 + \frac{1}{2}) \\
\text{for } x = x_0, \ldots, x_1 \text{ do} \\
\quad \text{draw}(x, y) \\
\quad \text{if } d < 0 \text{ then} \\
\quad \quad y \leftarrow y + 1 \\
\quad \quad d \leftarrow d + (y_0 - y_1) + (x_1 - x_0) \\
\quad \text{else} \\
\quad \quad d \leftarrow d + (y_0 - y_1)\
\]
Other cases: $0 \leq m \leq 1$
Other cases: $-1 \leq m \leq 0$
Other cases: $|m| > 1$