CS 130, Midterm 1

Solutions

1	2	3	4	5	6	7	8	9	10	11	12	13

Read the entire exam before beginning. **Manage your time carefully.** This exam has 50 points; you need 40 to get full credit. Additional points are extra credit.

Short answer. For each question below, provide a brief 1-3 sentence explanation.

Problem 1 (2 points)

(a) What is aliasing? (b) Describe a strategy that is used to avoid it. (We have discussed a few strategies in lecture; you may pick any of them.)

We have seen aliasing in a few contexts. (1) The jagged edges at the sides of smooth objects can be corrected (in ray tracing) by casting multiple rays per pixels (ideally sampled randomly). (2) The patterns that occur when rending a checkerboard is another type of aliasing, which can be solved in the same way. In the case of texture maps, mipmapping and filtering (linear/cubic interpolation) were also discussed as improvements.

Problem 2 (2 points)

Which of these features lead to recursion in a ray tracer? No explanation is required, but no partial credit is possible. (1) Texture mapping, (2) transparency shader, (3) Phong shader, (4) antialiasing, (5) reflective shader, (6) bump mapping, (7) area lights.

Only reflective and transparency shaders lead to recursion, since they recursively shade reflected (and possibly also transmitted) rays. Note that shadow rays are not recursive, so there is no recursion associated with lights or Phong shaders. Antialiasing and area lights involve casting more rays, but not recursively. Bump mapping and texture mapping just add image lookups to the shading calculation.

Problem 3 (2 points)

What are barycentric coordinates and what are they used for?

Barycentric coordinates are numbers that express the location of a point within a triangle. They are frequently used for interpolation (texture coordinates, colors, etc.) and for inside-outside queries.

Problem 4 (2 points)

Given two vectors $\vec{u}$ and $\vec{w}$, how do we determine whether the vectors are orthogonal?

$\vec{u} \cdot \vec{w} = 0$

Problem 5 (2 points)

Why does the intensity of a point light fall off with distance?

Imagine that the light is illuminating a dome centered at the light. The same amount of energy is emitted by the light no matter how big the dome is, but for a bigger dome that energy is spread over a larger surface area. The farther surface thus experiences less intense light.

Problem 6 (0 points)

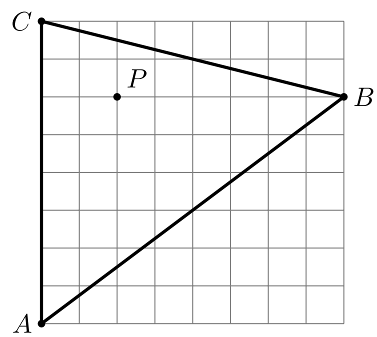
Provide a more efficient implementation of this function:

```
double foo(double x)
{
  return x*x*x*x;
}
```

```
double foo(double x)
{
  double y=x*x;
  return y*y;
}
```

Problem 7 (5 points)

Find the barycentric weights for the point P in the triangle below.



To do this, we need to compute the areas of the triangles, which we can then use to compute barycentric weights. Note that all of the triangles we need have an edge that is horizontal or vertical, so $A = \frac{1}{2}bh$ is easy to compute.

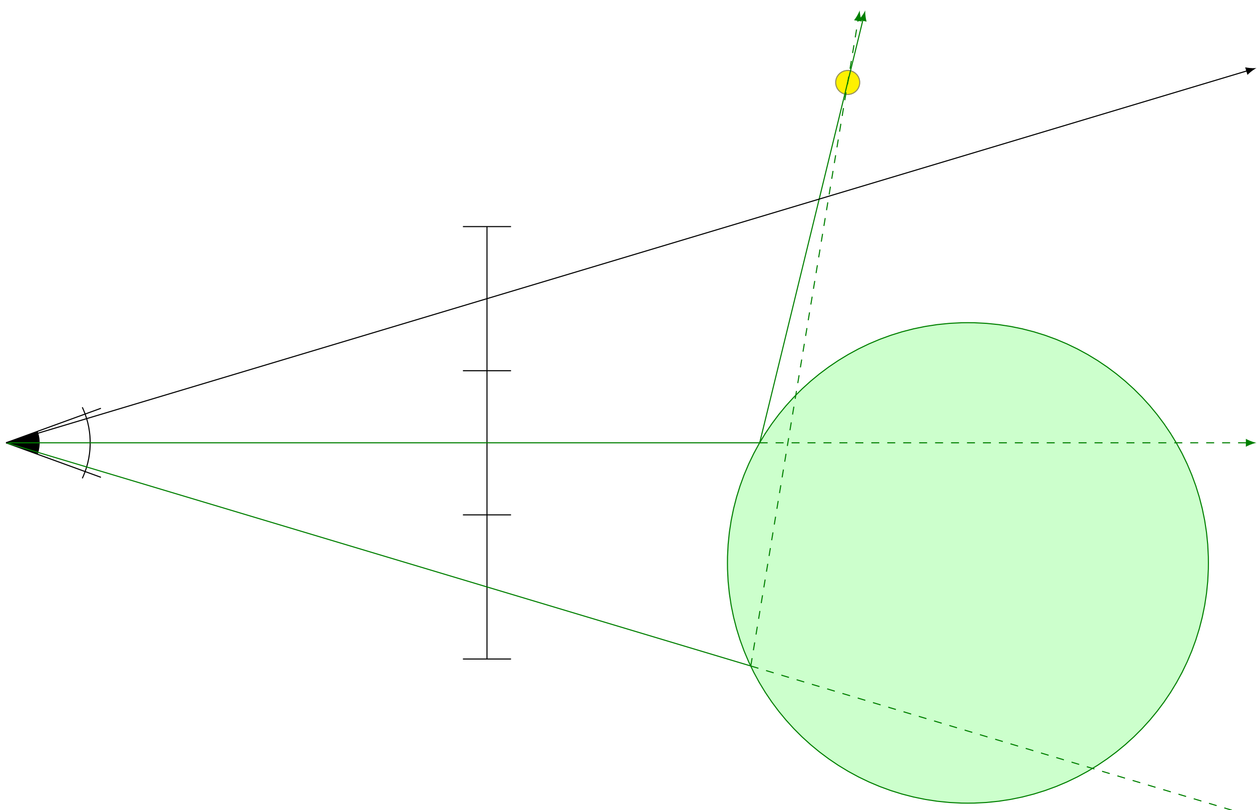
$$\text{area}(ABC) = 32 \qquad \text{area}(PBC) = 6 \qquad \text{area}(APC) = 8 \qquad \text{area}(ABP) = 18$$

$$\alpha = \frac{\text{area}(PBC)}{\text{area}(ABC)} = \frac{6}{32} \qquad \beta = \frac{\text{area}(APC)}{\text{area}(ABC)} = \frac{8}{32} \qquad \gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)} = \frac{18}{32}$$

Problem 8 (10 points)

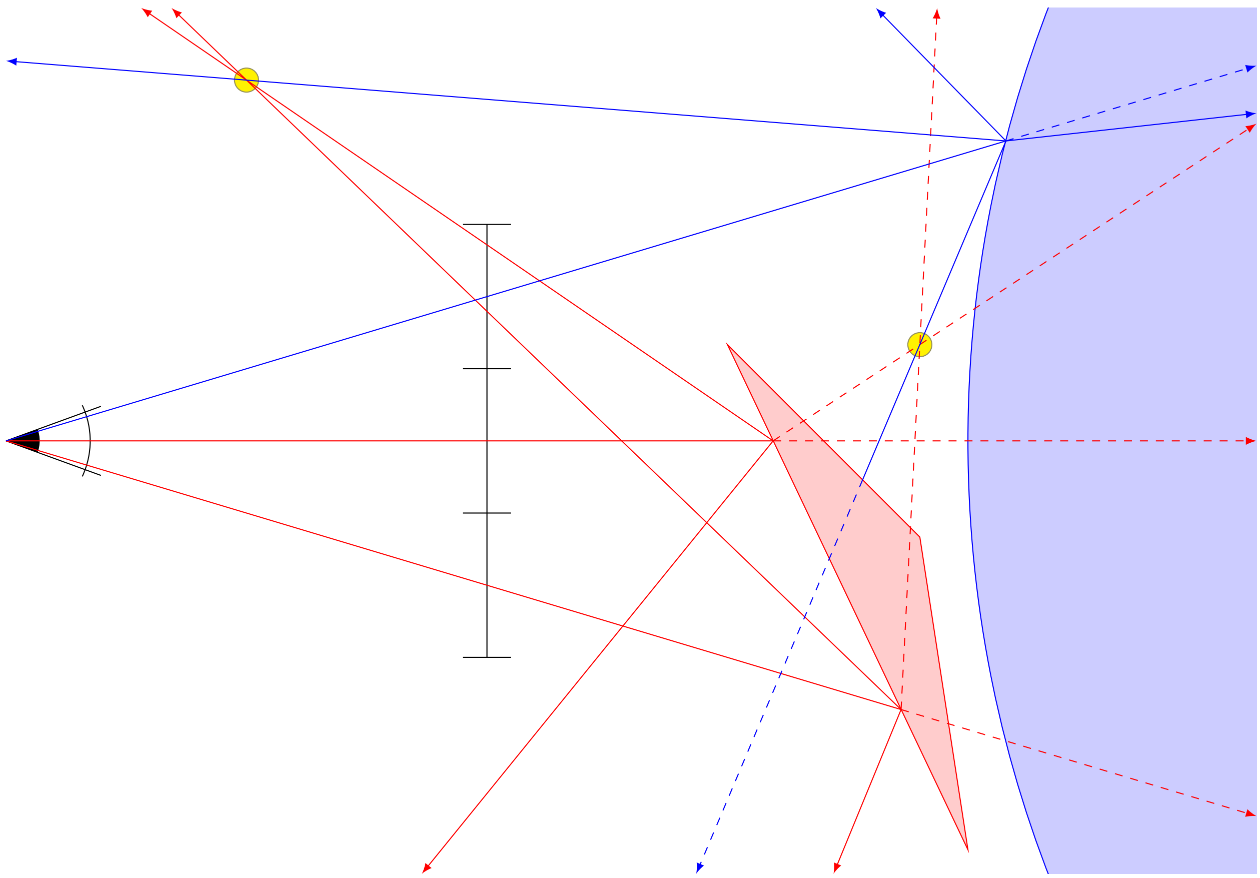
Below is a simple 2D raytracing setup. The 1D image has three pixels. The green object is made of wood.

The yellow circle is a point light. Draw all of the rays that would be cast while raytracing this scene.

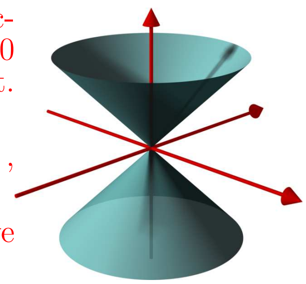


Problem 9 (10 points)

Below is a simple 2D raytracing setup. The 1D image has three pixels. The blue object is made of glass (reflective and transparent). The red object is reflective. The two yellow circles are point lights. Draw all of the rays that would be cast while raytracing this scene.



Problems 10-12 refer to the cone defined by the implicit function $f(x,y,z) = x^2 + y^2 - z^2$. You may assume that $f(x,y,z) > 0$ corresponds to *outside* of the cone. The cone is illustrated at right.



These problems also refer to the *ray* defined by $g(t) = \begin{pmatrix} t+1 \\ t \\ t+2 \end{pmatrix}$, where $t \geq 0$. All of these problems are independent (you can solve them in any order, even if you have not solved earlier ones).

Problem 10 (3 points)

What are the direction and endpoint of the ray?

Endpoint is $g(0) = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$. Direction is $\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. (Don't forget to normalize!)

Problem 11 (3 points)

Compute all intersection *locations* of the cone with the ray. [Hint: the answer should come out nicely; if it does not, check your math.]

Plugging $g(t)$ into $f(x,y,z)$ for the cone we get $(t+1)^2 + t^2 - (t+2)^2 = t^2 - 2t - 3 = (t-3)(t+1)$. Thus, $t = -1, 3$. Since this is a ray and $t \geq 0$, we can exclude the first. This leaves $t = 3$, which corresponds to $g(3) = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$.

Problem 12 (4 points)

What is the normal direction at an arbitrary point (x, y, z) lying on the surface of the cone? Don't worry about whether the normal points inwards or outwards.

$$f = x^2 + y^2 - z^2$$
$$\nabla f = \begin{pmatrix} 2x \\ 2y \\ -2z \end{pmatrix}$$
$$\|\nabla f\| = 2\sqrt{x^2 + y^2 + z^2}$$
$$n = \frac{\nabla f}{\|\nabla f\|} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \begin{pmatrix} x \\ y \\ -z \end{pmatrix}$$

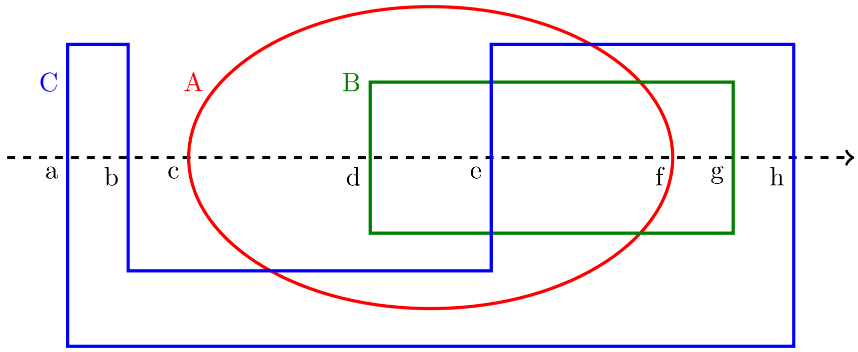
Note that $x^2 + y^2 - z^2 = 0$ so that $\sqrt{x^2 + y^2 + z^2} = \sqrt{2z^2} = |z|\sqrt{2}$.

An alternative strategy is to parameterize the surface, such as with (r, θ) and $\mathbf{w} = (x, y, z) = (r\cos\theta, r\sin\theta, r)$.

$$\mathbf{w} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ r \end{pmatrix}$$
$$\mathbf{w}_r = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$
$$\mathbf{w}_\theta = \begin{pmatrix} -r\sin\theta \\ r\cos\theta \\ 0 \end{pmatrix}$$
$$\mathbf{w}_r \times \mathbf{w}_\theta = \begin{pmatrix} (\sin\theta)0 - 1(r\cos\theta) \\ 1(-r\sin\theta) - (\cos\theta)0 \\ (\cos\theta)(r\cos\theta) - (\sin\theta)(-r\sin\theta) \end{pmatrix} = \begin{pmatrix} -r\cos\theta \\ -r\sin\theta \\ r \end{pmatrix} = \begin{pmatrix} -x \\ -y \\ z \end{pmatrix}$$
$$\|\mathbf{w}_r \times \mathbf{w}_\theta\|^2 = (-r\cos\theta)^2 + (-r\sin\theta)^2 + r^2 = 2r^2 = 2z^2$$
$$n = \frac{\mathbf{w}_r \times \mathbf{w}_\theta}{\|\mathbf{w}_r \times \mathbf{w}_\theta\|} = -\frac{1}{|z|\sqrt{2}} \begin{pmatrix} x \\ y \\ -z \end{pmatrix}$$

Problem 13 (5 points)

You are given the ray below (dashed) and objects A, B, and C. The intersections between the individual objects and the ray are labeled (a-h). For each of the Booleans composite objects below, identify the closest intersection point in the table. It is sufficient to write its label; no explanation is required. If no intersection occurs, write "x" as your intersection point. You must write your answer in the table for it to count.



#	object	answer
-	$A \cup B \cup C$	a
(a)	$A \cap B \cap C$	e
(b)	$A - (B \cup C)$	c
(c)	$B - (A \cup C)$	x
(d)	$B - (C - A)$	d
(e)	$B - A$	f
(f)	$B - C$	d