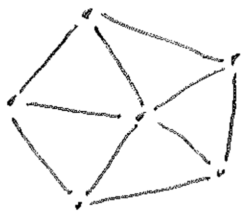
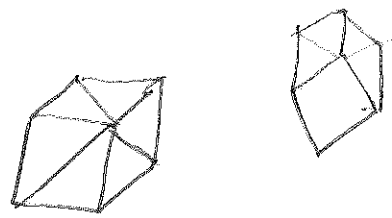
*

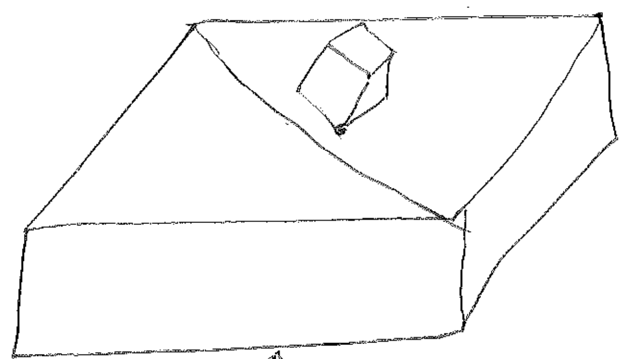


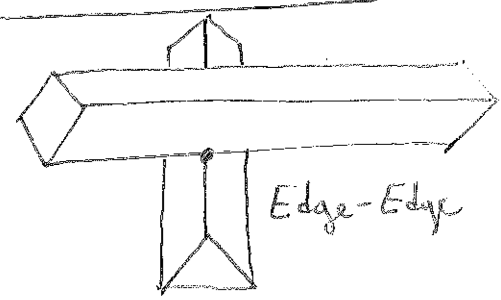
* collide with surface
* triangles

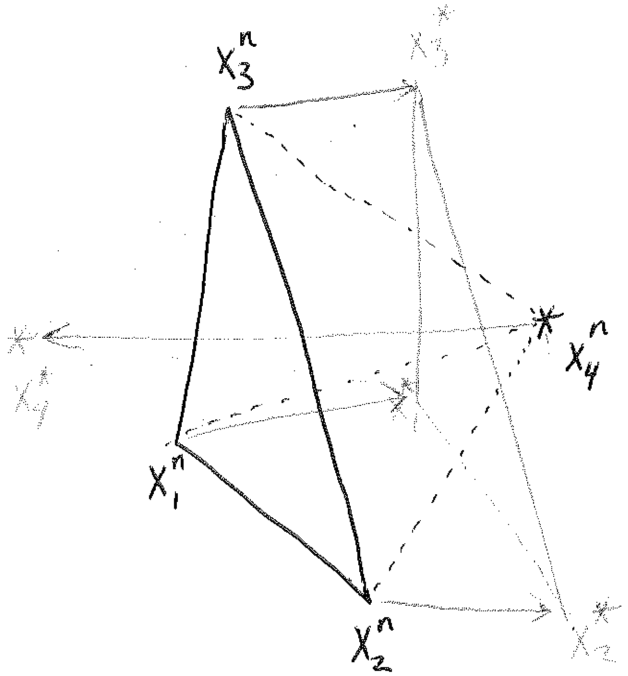


* maintain collision-free
* prevent/detect new ones
* make sure collision-free at end

Point-Triangle



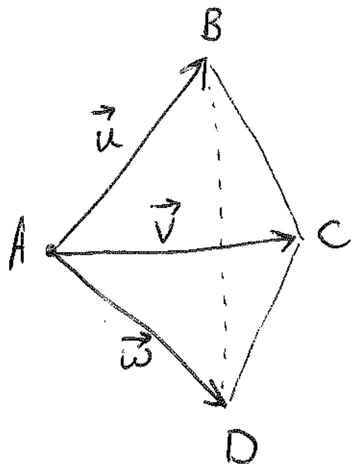


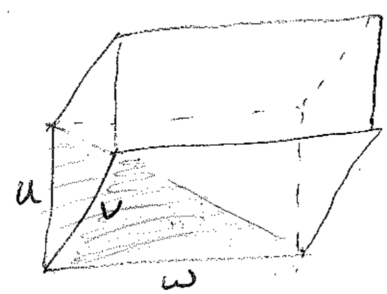


$$X_1(t) = X_1^n + t\,(X_1^* - X_1^n) \qquad t \in [0,1]$$
$$X_2(t) = X_2^n + t\,(X_2^* - X_2^n)$$
$$X_3(t) = \cdots \quad \vdots$$
$$X_4(t) = \cdots$$

* intersect when coplanar

* volume of tetrahedron $\Rightarrow 0$



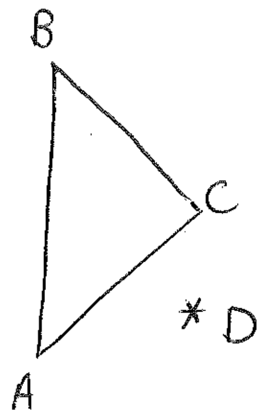


$\Rightarrow u \cdot (v \times w)$

volume

$$\text{volume} = \frac{1}{6}\vec{u}\cdot(\vec{v}\times\vec{w})$$
$$= \frac{1}{6}\underbrace{\left(X_2(t) - X_1(t)\right)}_{u} \cdot \left(\underbrace{\left(X_3(t) - X_1(t)\right)}_{v} \times \underbrace{\left(X_4(t) - X_1(t)\right)}_{w}\right)$$
$$= at^3 + bt^2 + ct + d \quad \left[\begin{array}{l} = 0 \\ \text{at intersection} \end{array}\right]$$

need root $t \in [0,1]$



barycentric weights

$$D = \alpha A + \beta B + \gamma C$$

inside if
$\alpha, \beta, \gamma \geq 0$

$$\gamma = 1 - \alpha - \beta$$

$$\alpha(A - C) + \beta(B - C) = D - C$$

$$\begin{pmatrix} A - C & B - C \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = D - C$$

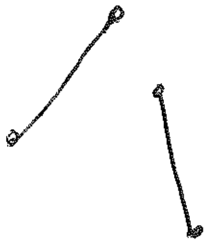
3x2 matrix   2-vec   3-vec

$$M \vec{z} = \vec{b}$$

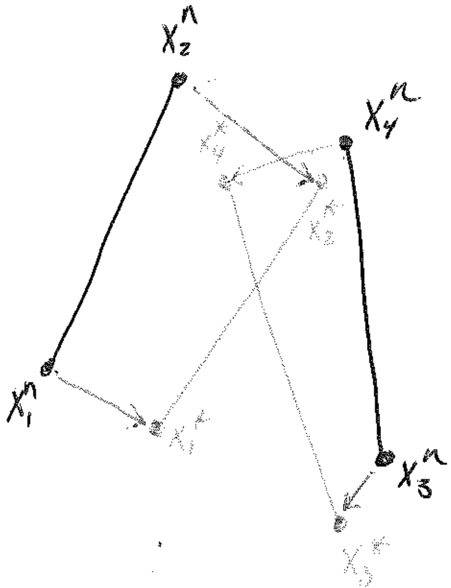
$$\underbrace{M^T M}_{2\times 2} z = M^T b$$

$$z = (M^T M)^{-1} M^T b$$

Edge - Edge



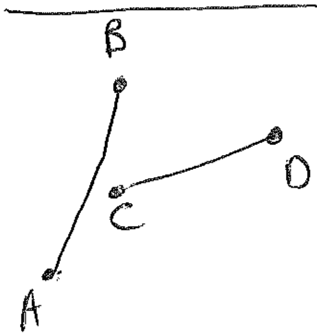
Assume straight trajectory



as before: $x_1(t) = \cdots$
$x_2(t) = \cdots$
$\vdots$

find t when $x_1\ x_2\ x_3\ x_4$ are <u>coplanar</u>

Same as before; solve cubic



$$z = A + \alpha(B-A) = C + \beta(D-C)$$

$$\alpha(B-A) + \beta(C-D) = C-A$$

$$\underbrace{\begin{pmatrix} B-A & C-D \end{pmatrix}}_{\substack{M \\ 3\times 2}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = C-A$$

$$M \vec{y} = \vec{b}$$

intersect if

$\boxed{\begin{array}{l} \alpha \in [0,1] \\ \beta \in [0,1] \end{array}}$

$$y = (M^T M)^{-1} M^T b$$