CS 230, Quiz 6

Solutions

You will have 6 minutes to complete this quiz. No books, notes, or other aids are permitted.

Problem 1

Construct a discretization for the Poisson equation ($\nabla^2 u = f$) in 2D to approximate $u(x, y)$ on $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. The function $f(x, y)$ is known in advance; you can evaluate it wherever you need it. Don't worry about boundary conditions or initial conditions. (Recall that $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.)

Let $x = i\Delta x - 1$ and $y = j\Delta y - 1$, with $0 \leq i \leq M$, $0 \leq j \leq N$, $\Delta x = \frac{2}{M}$, and $\Delta y = \frac{2}{N}$. Then, I can define my degrees of freedom as $u_{i,j} = u(x, y)$ and my right hand side as $f_{i,j} = f(x, y)$. Discretizing using a central difference gives

$$\begin{aligned}
\nabla^2 u &= f \\
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= f \\
\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} &= f_{i,j}
\end{aligned}$$