$$\times \quad \boxed{\frac{R^{n+1}-R^n}{\Delta t} = (\omega^*)^n R^n}$$

$$R^{n+1} = R^n + \Delta t \, (\omega^*)^n R^n \leftarrow = \underbrace{\Big(\overset{\text{identity}}{\delta} + \Delta t \, (\omega^n)^*\Big)}_{\text{replace with } \underline{\text{rotation}}} R^n$$

$$= R_e^{n+1} + O(\Delta t^2)$$

replace with $R^{n+1} = g(\Delta t \, \omega^n) R^n$

$$\rightarrow g(u) g(u)^T = \delta$$

$$g(\Delta t \, u) = \delta + \Delta t \, u^* + O(\Delta t^2)$$

$$g(u) = \exp(u^*) = \delta + u^* + \frac{1}{2}(u^*)^2 + \frac{1}{6}(u^*)^3 + \frac{1}{24}(u^*)^4 + \dots$$

$$\boxed{\theta = \|u\| \quad z = \frac{u}{\|u\|}}$$

$$\boxed{g(u) = \delta + (\sin\theta) z^* + (1-\cos\theta)(z^*)^2}$$

$$\sin\theta \approx \theta - \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5 - \dots$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots$$

$$g(\Delta t \, \omega) = \delta + \sin\|\Delta t \, \omega\| \left(\frac{\Delta t \, \omega}{\|\Delta t \, \omega\|}\right)^* + (1-\cos\|\Delta t \, \omega\|)\left[\left(\frac{\Delta t \, \omega}{\|\Delta t \, \omega\|}\right)^*\right]^2$$

$$\approx \delta + \Delta t \, \omega^* + O(\Delta t^3) + O(\Delta t^2)$$

$$\cos\theta \approx 1 - \frac{1}{2}\theta^2 + \frac{1}{24}\theta^4 - \dots$$

Fluids

$F = m\underline{a}$

$\vec{u}$ is velocity

$\vec{u}(x, y, z, t)$

$\frac{\partial u}{\partial t}$ ← partial

$$m\frac{Du}{Dt} = f = \underbrace{f_g}_{\text{gravity}} + \overbrace{f_p}^{\text{pressure}} + \underbrace{f_v}_{\text{viscosity}}$$

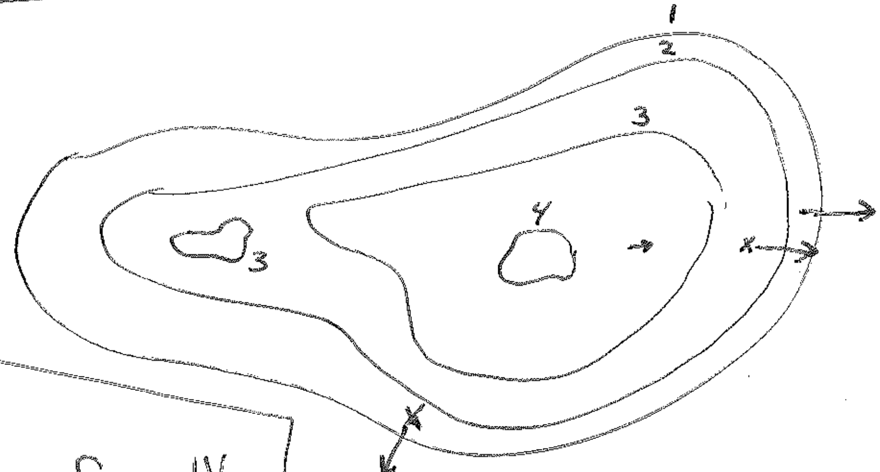
↑ material derivative

gravity $f_g = mg$

Pressure

$f_p = -\nabla p \, V$

↑ volume



$$f_p = \int_{\partial\Omega} -p\vec{n}\, dA$$

$\partial\Omega$ ↑ boundary

$$= -\int_{\partial\Omega} p \, d\vec{S} = -\int_{\Omega} \nabla p \, dV$$

gradient theorem

direction normal to contours

proportional to rate of pressure change

$f_p \approx -\nabla p \, V$