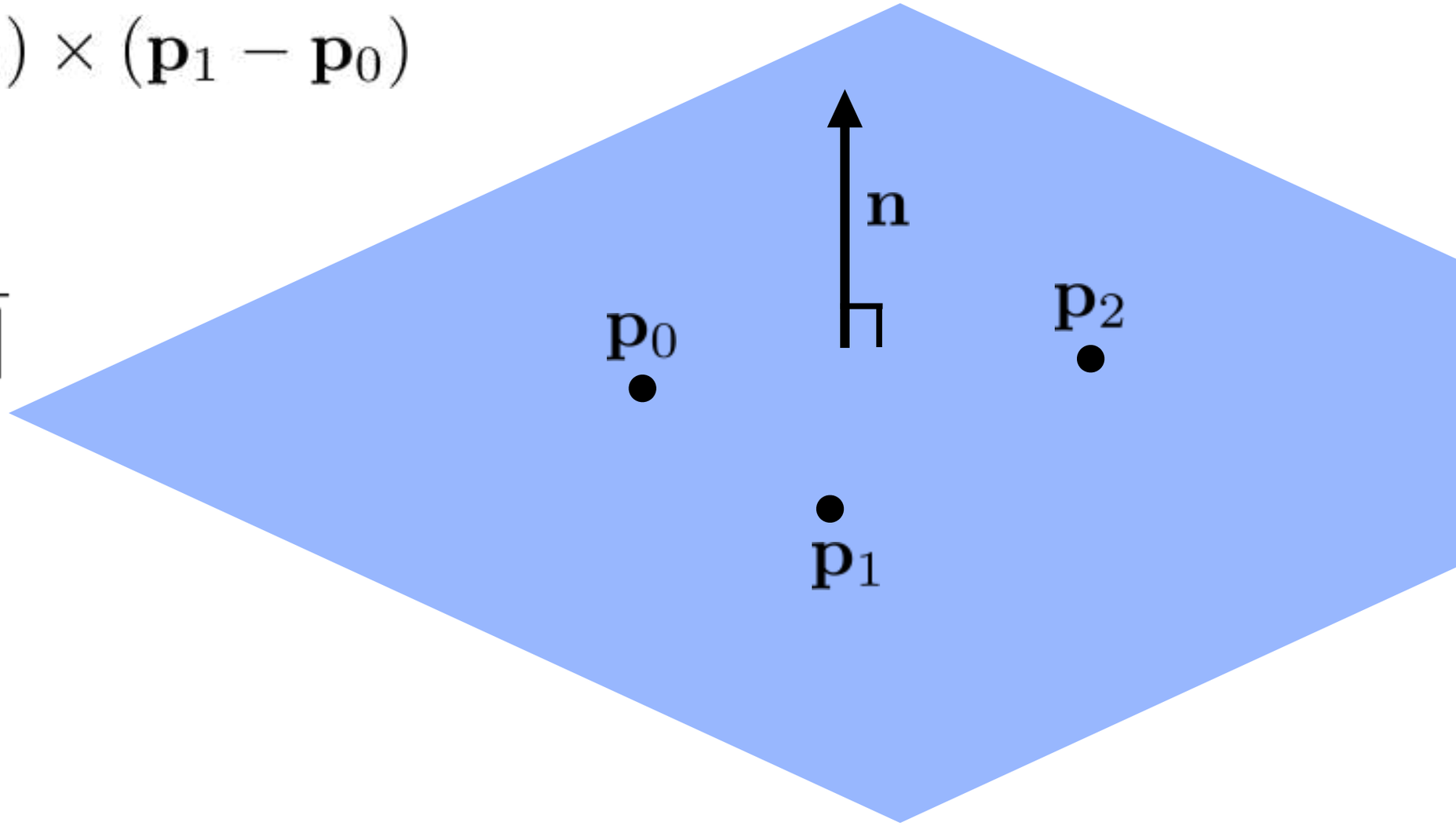
Computing Normal Vectors

Plane Normals

$$\mathbf{v} = (\mathbf{p}_2 - \mathbf{p}_0) \times (\mathbf{p}_1 - \mathbf{p}_0)$$

$$\mathbf{n} = \frac{\mathbf{v}}{||\mathbf{v}||}$$



Implicit function normals

$$f(\mathbf{p}) = 0$$

$$\nabla f(\mathbf{p})$$

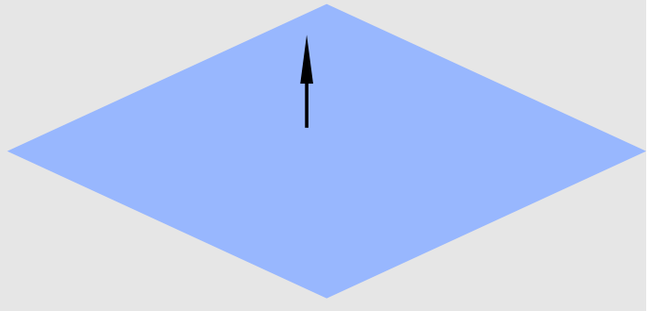
$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$$

sphere

$$\mathbf{p} \cdot \mathbf{p} - r^2 = 0$$

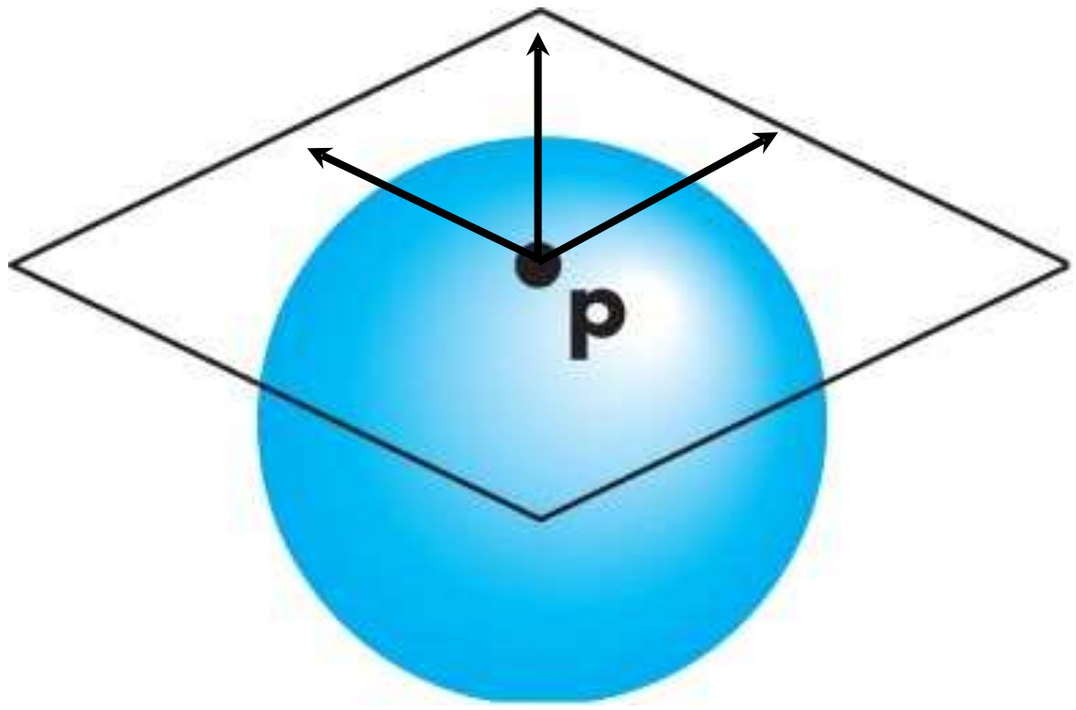
plane

$$\mathbf{n} \cdot (\mathbf{p} - \mathbf{p}_0) = 0$$



Parametric form

$$\mathbf{p}(u,v) = \begin{pmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{pmatrix}$$



tangent vectors $\dfrac{\partial \mathbf{p}}{\partial u} \qquad \dfrac{\partial \mathbf{p}}{\partial v}$

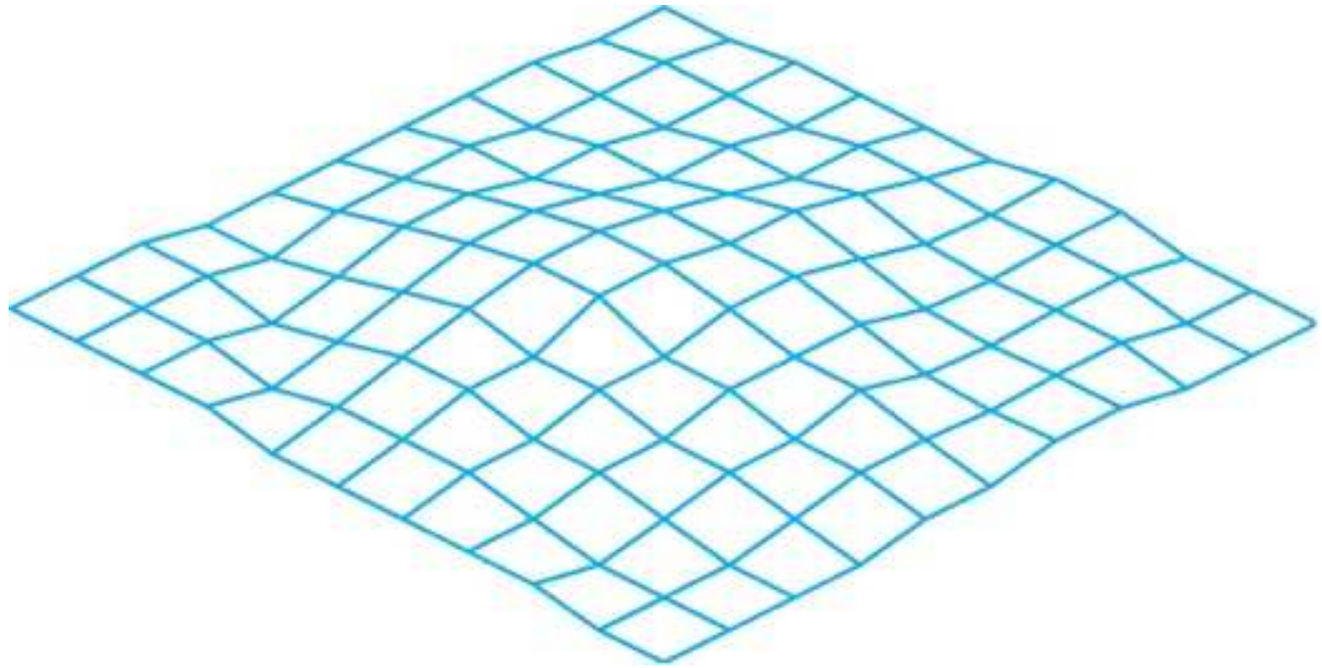
normal $$\frac{\frac{\partial \mathbf{p}}{\partial u} \times \frac{\partial \mathbf{p}}{\partial v}}{||\frac{\partial \mathbf{p}}{\partial u} \times \frac{\partial \mathbf{p}}{\partial v}||}$$

Shading Polygonal Geometry

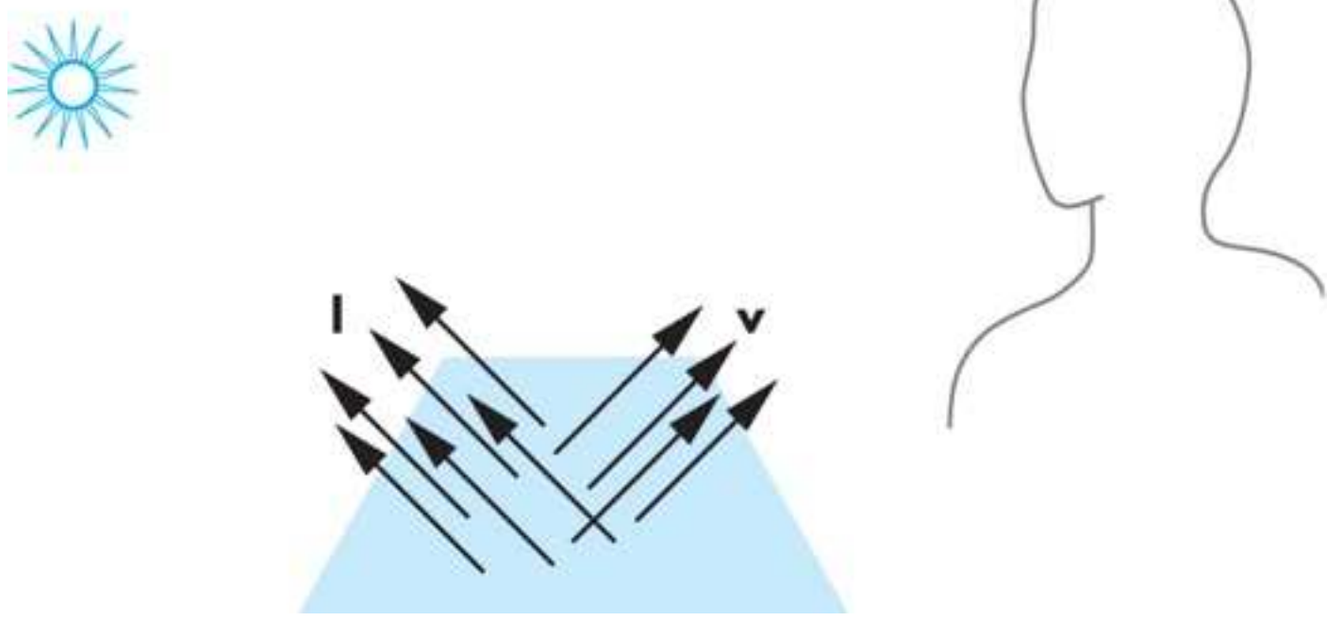
Smooth surfaces are often approximated by polygons

Shading approaches:

1. Flat
2. Smooth (Gouraud)
3. Phong



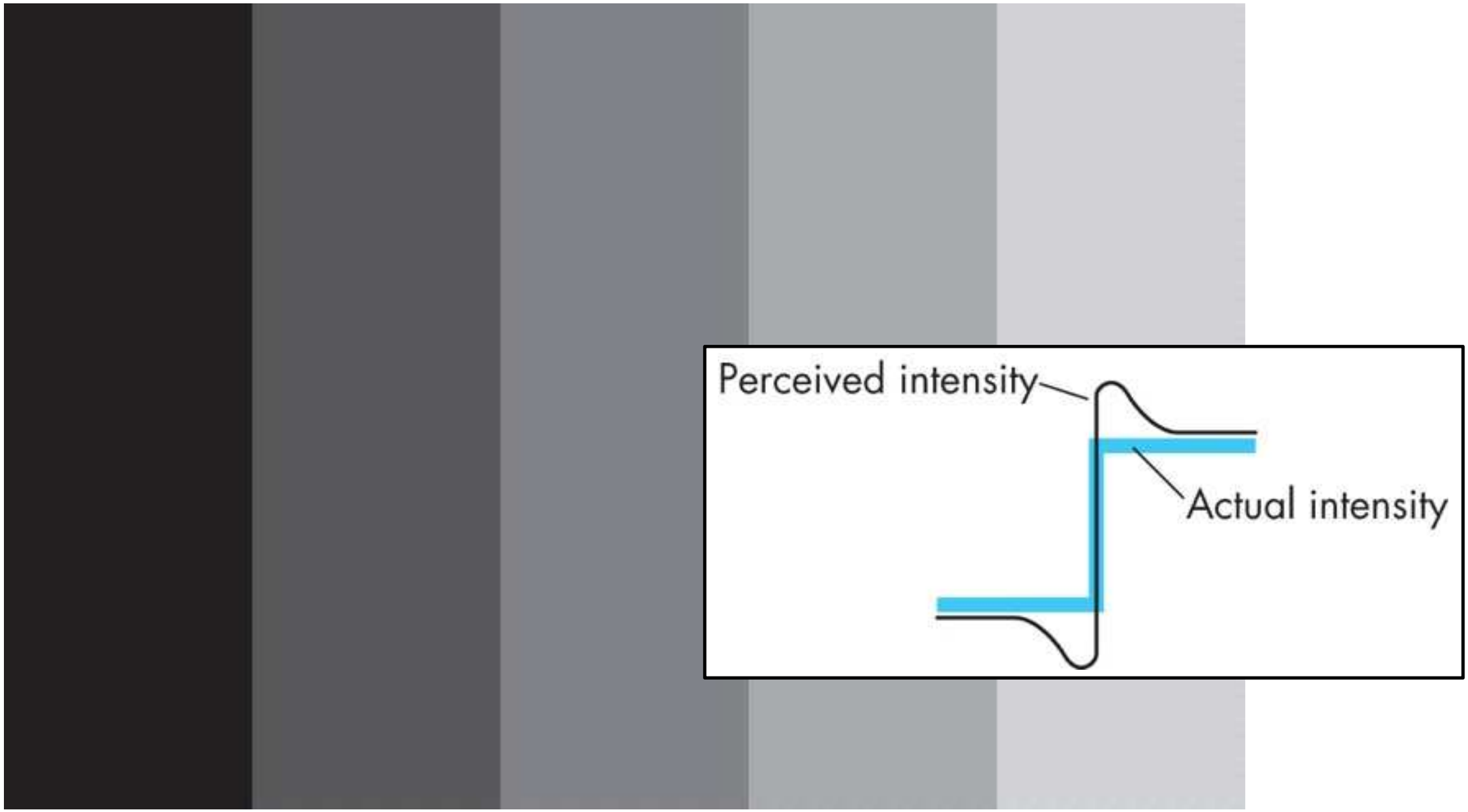
Flat Shading



do the shading calculation once per **polygon**

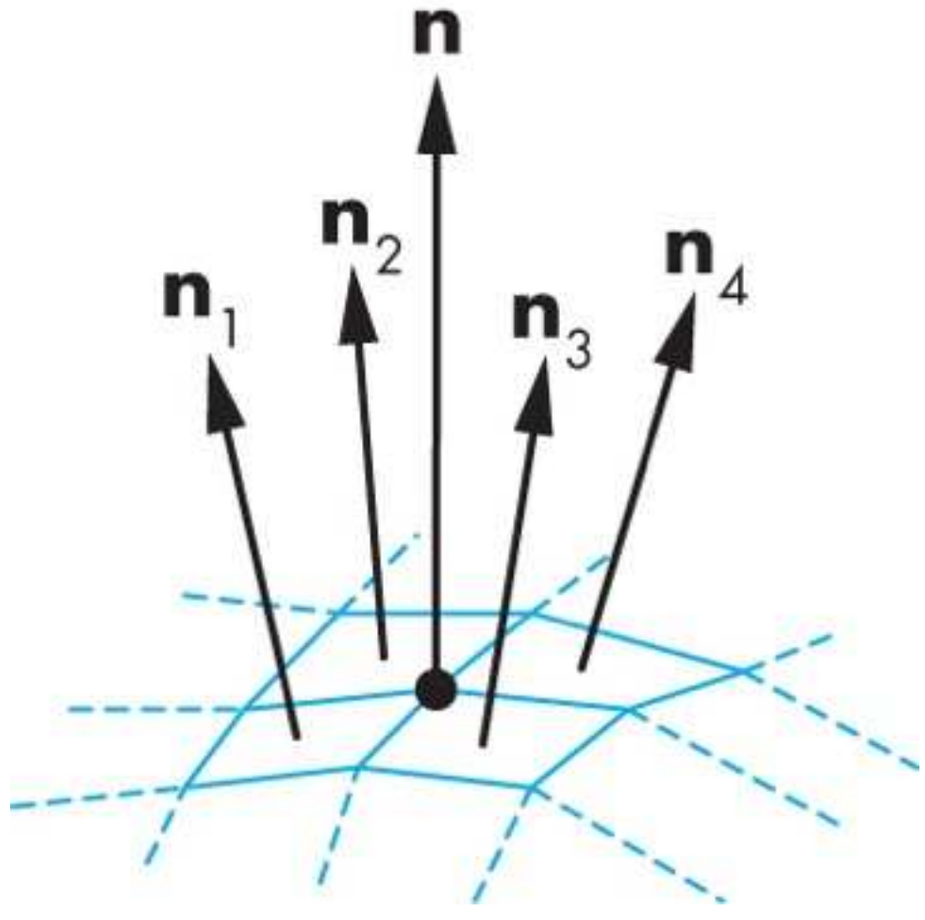
valid for light at ∞
and viewer at ∞
and faceted surfaces

Mach Band Effect



Smooth Shading

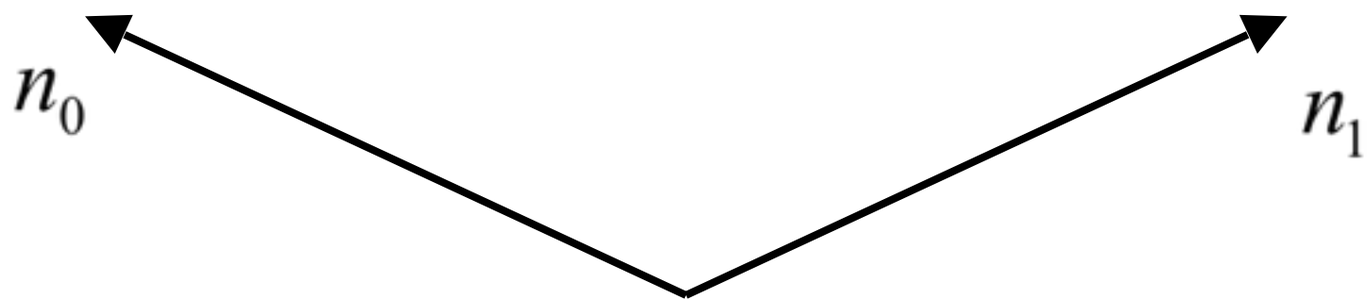
$$\mathbf{n} = \frac{\mathbf{n}_1 + \mathbf{n}_2 + \mathbf{n}_3 + \mathbf{n}_4}{||\mathbf{n}_1 + \mathbf{n}_2 + \mathbf{n}_3 + \mathbf{n}_4||}$$



do the shading calculation once per **vertex**

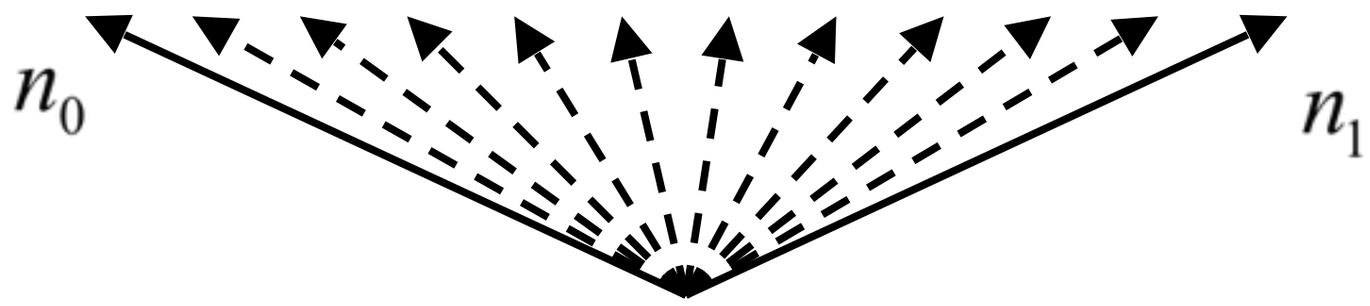
Interpolating Normals

- Must renormalize



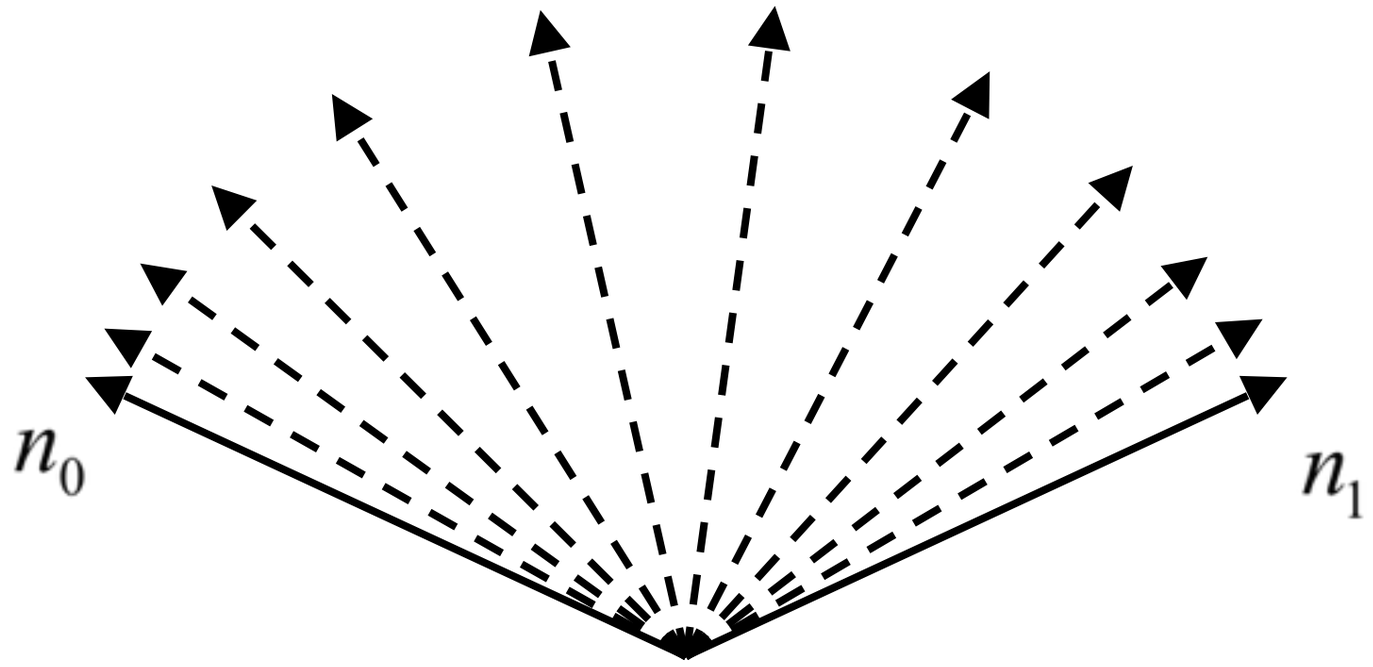
Interpolating Normals

- Must renormalize

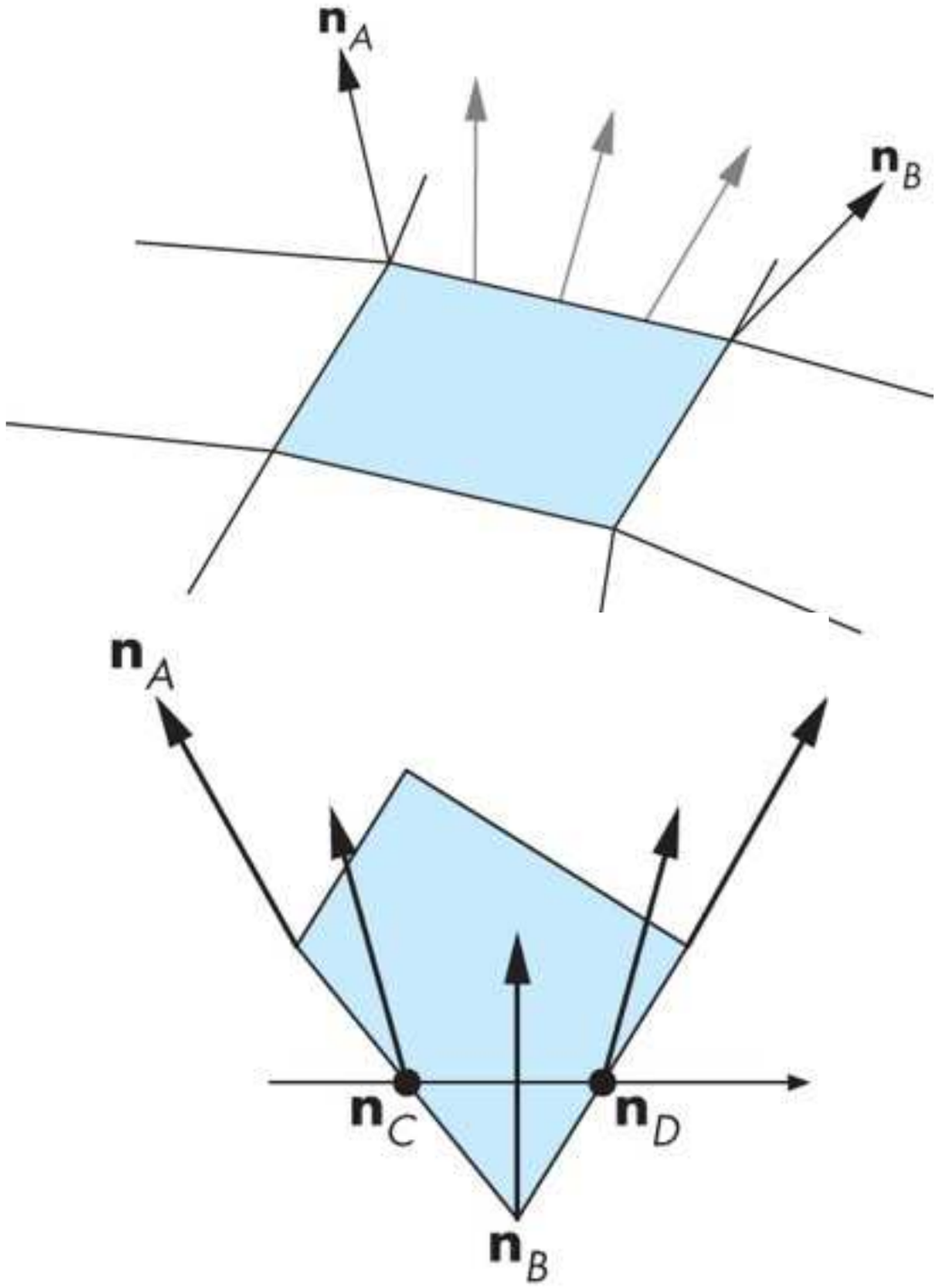


Interpolating Normals

- Must renormalize



Phong Shading

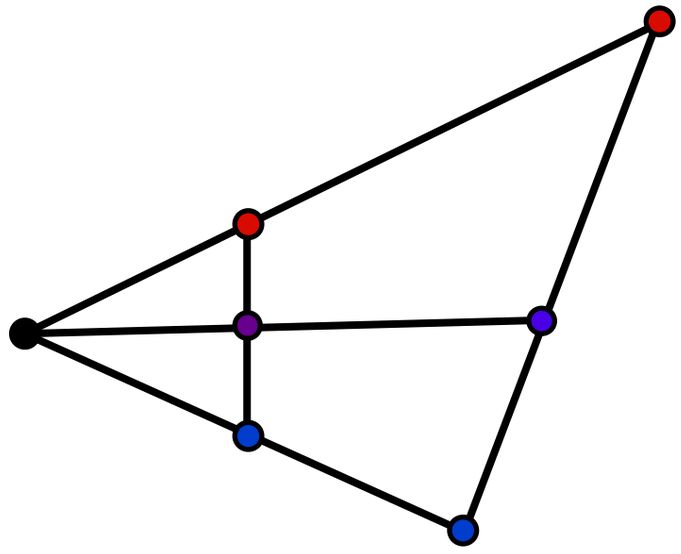


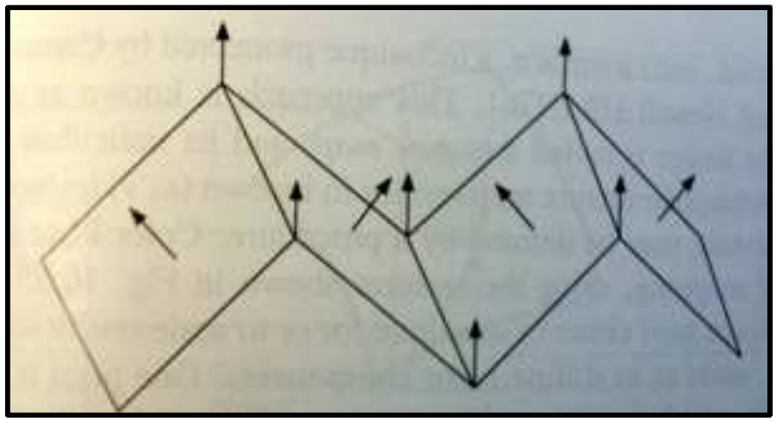
do the shading calculation once per **fragment**

Comparison

Problems with Interpolated Shading

- Polygonal silhouette
- Perspective distortion
- Orientation dependence
- Unrepresentative surface normals





[Foley, van Dam, Feiner, Hughes]