Heat: $\frac{\partial u}{\partial t} - a \nabla^2 u = 0$

1D: $\frac{\partial u}{\partial t} - a \frac{\partial^2 u}{\partial x^2} = 0$ $\qquad 0 \le x \le 1$

$0 \le t \le T$

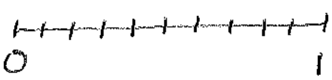
Initial: $u(x,0) = f(x)$

Boundary: $u(0,t) = g(t)$

$u(1,t) = h(t)$

$t = 0, \Delta t, 2\Delta t, 3\Delta t, \ldots, T \qquad \Delta t = \frac{T}{N}$

$x = 0, \Delta x, 2\Delta x, 3\Delta x, \ldots, 1 \qquad \Delta x = \frac{1}{M}$



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u_i^n ← time ← space

$u_i^0 = f(i \Delta x)$ ← initial

$u_0^n = g(n \Delta t) \qquad u_M^n = h(n \Delta t)$

$$\frac{\partial u}{\partial t} - a \frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{\partial u}{\partial t} \approx \frac{u_i^{n+1} - u_i^n}{\Delta t}$$

$$\frac{\partial u}{\partial x} = \frac{u_{i+1}^n - u_i^n}{\Delta x} \quad \text{or} \quad \frac{\partial u}{\partial x} = \frac{u_i^n - u_{i-1}^n}{\Delta x}$$

forward difference $\qquad$ backward difference

$$\frac{\partial u}{\partial x} = \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x}$$

central difference

$$\frac{\partial^2 u}{\partial x^2} = \frac{\dfrac{u_{i+1}^n - u_i^n}{\Delta x} - \dfrac{u_i^n - u_{i-1}^n}{\Delta x}}{\Delta x} = \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

$$\frac{\partial^3 u}{\partial x^3} = \pm \frac{u_{i+2}^n - 3u_{i+1}^n + 3u_i^n - u_{i-1}^n}{\Delta x^3}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} - a\,\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} = 0 \qquad \text{FTCS}$$

forward time
central space

Solve for u_i^{n+1}

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} - a\,\frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{\Delta x^2} = 0 \qquad \text{BTCS}$$

$u(x,y,0) = f(x,y)$

$u(0,y,t) = g(y,t)$

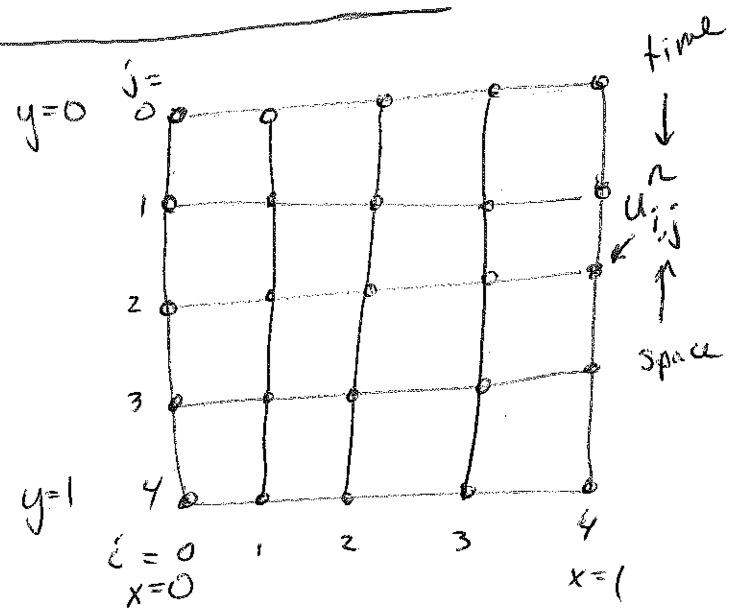
$u(1,y,t) = h(y,t)$

$u(x,0,t) = k(x,t)$

$u(x,1,t) = m(x,t)$

$u_{i,j}^0 = f(i\Delta x, j\Delta y)$

$u_{0,j}^n = g(j\Delta y, n\Delta t)$



$x = i\Delta x$

$y = j\Delta y$

$$2D: \quad \frac{\partial u}{\partial t} - a\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$$

FTCS

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} - a\left(\frac{u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}}{\Delta x^2} + \frac{u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}}{\Delta y^2}\right) = 0$$

Conjugate Gradient

Sparse Solver