Scale: $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \longrightarrow \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$

2D

3D

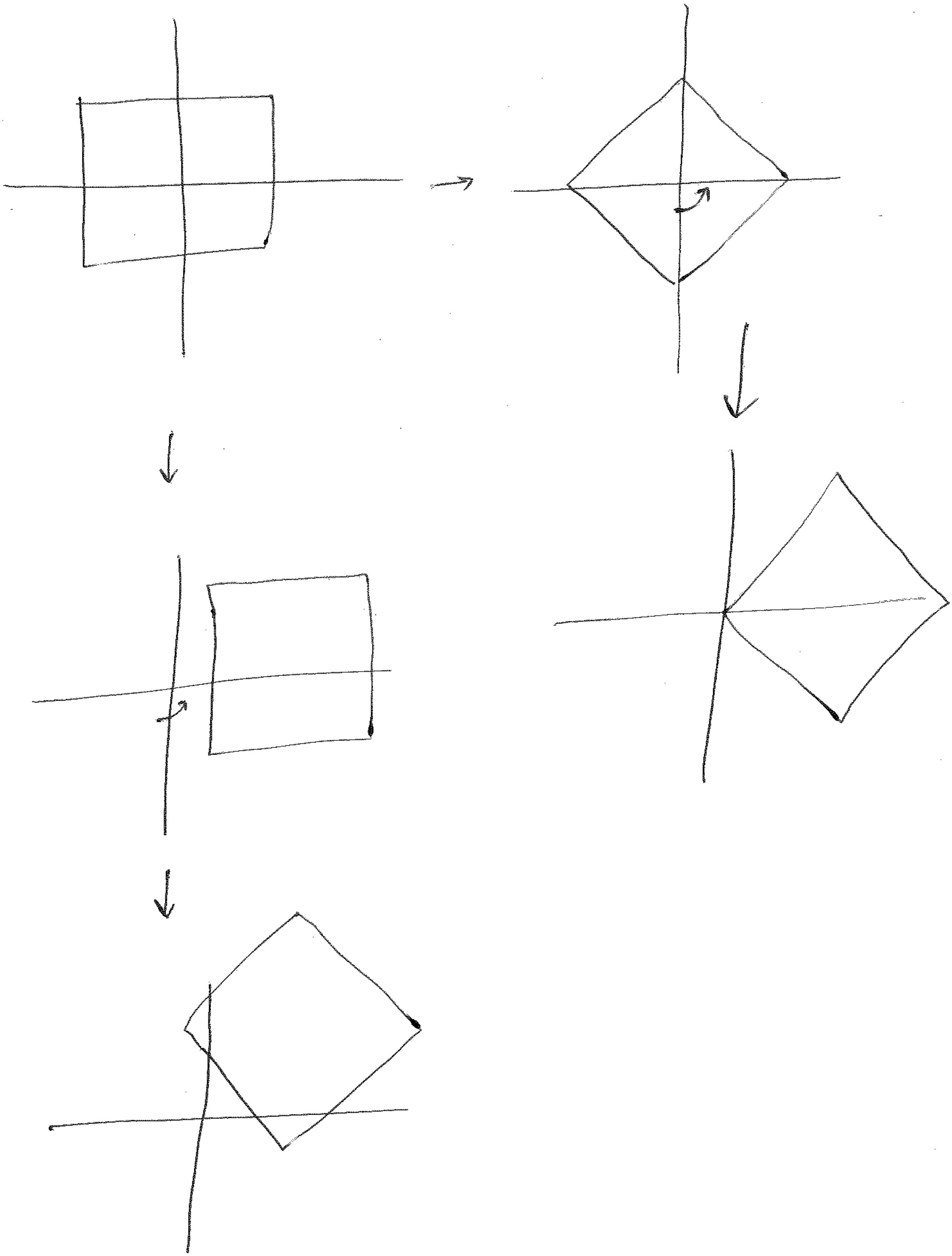
$$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Homogeneous Coordinates

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \longrightarrow \begin{pmatrix} x/w \\ y/w \\ z/w \end{pmatrix}$$

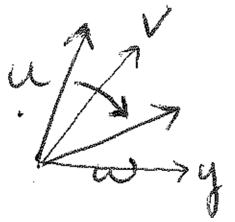
$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \equiv \begin{pmatrix} ax \\ ay \\ az \\ aw \end{pmatrix}$$

Do not commute:



want to show $R^T R = I$ or $R R^T = I$

or $R^T = R^{-1}$



$w = Ru$

$y = Rv$

$\|w\| = \|u\|$

$\|y\| = \|v\|$

$w \cdot y = \|w\| \, \|y\| \cos\theta$

$u \cdot v = \|u\| \, \|v\| \cos\theta$

$w \cdot y = u \cdot v$

$u^T v = w^T y = (Ru)^T R v$

$= u^T (R^T R) v$

$$u^T \underbrace{(I - R^T R)}_{A} v = 0$$

$u^T A v = 0$ any u, v

$= A_{ik}$ $\quad u = e_i = \begin{array}{cc} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \leftarrow i \end{array}$

$v = e_k$

$A = 0$

$\Downarrow$

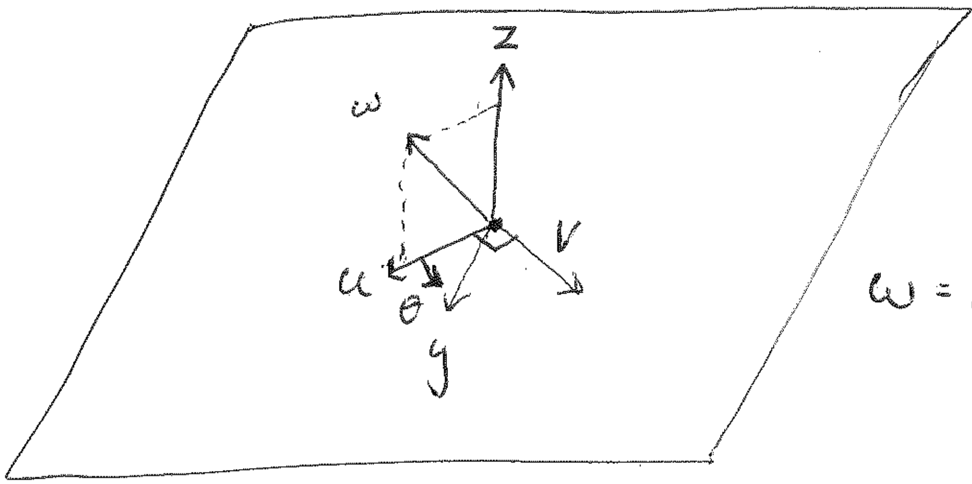
$\boxed{R^T R = I}$

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ reflection

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}^T \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

rotation: $\det(R) = 1$

R rotate around $\vec{z}$ by θ. $\|\vec{z}\| = 1$



$Rz = z$

$w = a\vec{z} + \vec{u}$

$\vec{z}\cdot\vec{u} = 0$

$y = Ru$

$= bu + cv$

$v = z \times u$

$= z \times w - \underbrace{(z \times z)}_{0}(w\cdot z)$

$= z \times w$

$\|v\| = \|z\|\,\|u\|\,\underbrace{\sin\underbrace{\phi}_{\frac{\pi}{2}}}_{1} = \|u\|$

$w' = Rw = Raz + Ru$

$= az + \underbrace{Ru}_{y}$

$w\cdot z = a\underbrace{z\cdot z}_{1} + \underbrace{u\cdot z}_{0}$

$a = w\cdot z$

$w = (w\cdot z)z + u$

$u = w - (w\cdot z)z$

$\|y\| = \|u\|$

$y\cdot u = \|y\|\,\|u\|\cos\theta$

$b(u\cdot u) + c\underbrace{(v\cdot u)}_{0} = (u\cdot u)\cos\theta$

$b = \cos\theta$

$u\cdot u = y\cdot y = b^2(u\cdot u) + 2bc\underbrace{(u\cdot v)}_{0} + c^2(v\cdot v)$

$(1 - b^2)\,u\cdot u = c^2(v\cdot v)$

$\sin^2\theta\,(u\cdot u) = c^2(v\cdot v) = c^2(u\cdot u) \Rightarrow c = \pm\sin\theta$ want (+)

$R\omega =$

$$\omega' = az + Ru$$

$$= (z \cdot \omega) z + y$$

$$= (z \cdot \omega) z + bu + cv$$

$$= (z \cdot \omega) z + (\cos\theta)(\omega - (z \cdot \omega) z) + (\sin\theta)(z \times \omega)$$

$$\boxed{R = zz^T + (\cos\theta)(I - zz^T) + (\sin\theta) z^*}$$

$$\boxed{z^*}\omega = z \times \omega$$