CS 141, Fall 2018 Assignment 5

Posted: November 25th, 2018 Due: December 9th, 2018, 11:59pm

Problem 1. (33 Points) You are given a set of cities, along with the pattern of highways between them in the form of an undirected graph $G = (V, E)$. Each stretch of highway $e \in E$ connects two of the cities, and you know its length in miles, l_e. You want to get from city s to city t. There is one problem: your car can only hold enough gas to cover L miles. There are gas stations in each city, but not between cities. Therefore, you can only take a route (path) if every one of its edges has length $l_e \leq L$.

1. Given the limitation of you cars fuel tank capacity, show how to determine in linear time whether there is a feasible route from s to t.
2. You are planning to buy a new car, and you want to know the minimum rule tank capacity that is needed to travel from s to t. Give a $O((n + m)logn)$ algorithm to determine this.

Hint for (2): Modify Dijkstras algorithm to find paths that minimize the maximum weight of any edge on the path (instead of the path length).

Problem 2. (33 Points) Consider the following algorithm for MST, called Reverse-Delete. Start with the full graph $G = (V, E)$ and begin deleting edges in order of decreasing cost. As we get to each edge e (starting from the most expensive), we delete it as long as doing so would not actually disconnect the graph we currently have. Prove that Reverse-Delete computes the optimal MST.

Problem 3. (34 Points) Given a graph $G = (V,\ E)$, find the minimum number of colors to mark all nodes in V so that no two adjacent nodes have the same color. Hint: You can solve this problem by finding the maximum clique in G.

The following problems are optional. You can get full credit without submitting them. Solving them could give you extra credit.

Problem 4. (25 Points) (Optional Problem) Consider the matrix chain multiplication problem that is discussed in class. Does this problem belongs to complexity class *P*, *NP*, or none of them? Prove your answer.

Note: To prove belonging to *P* class, you need to provide a polynomial time *solution* algorithm pseudo code and analyze its complexity. To prove belonging to *NP* class, you need to provide a polynomial time *verification* algorithm pseudo code and analyze its complexity.

Problem 5. (25 Points) (Optional Problem) **Vertex Cover Problem**. A vertex cover set of a graph $G=(V,E)$ is a set of vertices $V_1 \in V$ such that each edge $e \in E$ is incident to at least one vertex in V_1. If edge $e = (u, v)$ connects two nodes u and v, e said to be an incident to u and an incident to v. The size of a vertex cover set V_1 if the number of vertices in the set ($|V_1|$). Prove that finding a vertex cover set of size k is an NP problem.

Problem 6. (25 Points) (Optional Problem) Let Q and R be two problems not known to be in NP. R is polynomial-time reducible to MAX-CLIQUE problem and MAX-CLIQUE problem is polynomial-time reducible to Q. Circle the correct answer(s) of the following questions:

a. Is R NP-hard?	Yes	NO	Do not know
b. Is Q NP-hard?	Yes	NO	Do not know
c. Is R NP-complete?	Yes	NO	Do not know
d. Is Q NP-complete?	Yes	NO	Do not know