

Problem 1. (10 points) Write an algorithm to construct the actual solution of the matrix chain multiplication problem (i.e., the parentheses order). Trace its output on the following examples:

- (a) Three matrices (A, B, and C) with dimensions $10 \times 50 \times 5 \times 100$, respectively.
- (b) Four matrices (A, B, C, and D) with dimensions $20 \times 5 \times 10 \times 30 \times 10$, respectively.

Problem 2. (20 points) Given two strings A and B and the following operations that can be performed on A. Find minimum number of operations required to make A and B equal.

1. Insert
2. Delete
3. Replace

Problem 3. (25 points) Let $A = \{a_1, a_2, \dots, a_n\}$ and be a set of n positive integer and let T be another integer. Design a dynamic programming algorithm that determines whether there exists a subset of A whose total sum is exactly T . Analyze the *time*– and *space*–complexity of your solution.

For instance, if $A = \{4, 5, 17, 23, 11, 2\}$ and $T = 35$ the algorithm should return True because the subset $\{5, 17, 11, 2\}$ sums to 35. For the same set of numbers if we choose $T = 31$ the problem has no solution, and the algorithm will return False.

Problem 4. (25 points) Let A be a $n \times m$ matrix of 0's and 1's. Design a dynamic programming $O(nm)$ time algorithm for finding the largest square block of A that contains 1's only.

Hint: Define the dynamic programming table $l(i, j)$ be the length of the side of the largest square block of 1's whose bottom right corner is $A[i, j]$.

Problem 5. (20 points) Given n dice each with m faces, numbered from 1 to m , find the number of ways to get sum X , where X is the sum of values on each face when all the dice are thrown.