Math 142-2, Project 3

Both partners' names here

Problem 1

In Project 2, we constructed a model for the change in a single gene in a population over time. If the gene has two variants, r and R, then three distinct types of individuals may exist in the population: RR, Rr, and rr. $A(t)$ is the number of individuals with genotype RR. $B(t)$ is the number of individuals with genotype Rr. $C(t)$ is the number of individuals with genotype rr. The model is then

$$\begin{aligned}\frac{dA}{dt} &= \frac{(A+B/2)^2}{A+B+C}(m-n(A+B+C))-aA\\ \frac{dB}{dt} &= \frac{2(C+B/2)(A+B/2)}{A+B+C}(m-n(A+B+C))-bB\\ \frac{dC}{dt} &= \frac{(C+B/2)^2}{A+B+C}(m-n(A+B+C))-cC\end{aligned}$$

To analyze this model, we will use a computer to numerically approximate the solution to this system of ODE's. Use fourth order Runge-Kutta to approximate the ODE. You may use any programming language you like. Use this implementation to explore the following by running simulations with a variety of initial conditions and parameters. In each case below, determine what affect the parameters have on the long term behavior of the population. Does the fraction of R allele, $g(t) = \frac{A(t)+B(t)/2}{A(t)+B(t)+C(t)}$, change over time? Does it approach some fixed value (such as 0 (R dies off), $\frac{1}{2}$ (equality), or 1 (r dies off))? How does this depend on the initial populations? For each problem, turn in plots for at least four sets of parameters and initial conditons which together tell a convincing story of what the population is doing. In each plot, show $A(t)$, $B(t)$, and $C(t)$ over time.

(a) $a = b = c$.
(b) $a = c$ but $b < a$. (heterozygote advantage).
(c) $a = c$ but $b > a$. (heterozygote disadvantage).
(d) $a = b$ but $c < a$. (rr offers some advantage).
(e) $a = b$ but $c > a$. (rr offers some disadvantage).
(f) $c > a > b$. This can occur with recessive traits, where the recessive trait rr is harmful, but yet the trait offers heterozygote advantage. How do the results differ qualitatively from those of case (e)?

Your solution goes here

Problem 2

Sickle-cell and Cystic fibrosis are two fairly common genetic diseases. Both are recessive and, without significant medical intervention, both are effectively fatal. Sickle-cell is known to confer heterozygote advantage (such individuals fare better when infected with malaria). Cystic fibrosis has no known heterozygote advantage. Yet even though no such advantage has been found, it is nevertheless hypothesized

that such an advantage must exist. Why?

Your solution goes here

Problem 3

The model proposed in these solutions has a major problem when the population is over its carrying capacity and one of the populations is zero. What is this problem? Suggest a modification to the model that would correct this.

Your solution goes here