Math 142-2, Project 3

Your name here

Problem 1

Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$. Show that if $\rho(x, 0) = \rho_{\max} - \rho(a - x, 0)$ then $\rho(x, t) = \rho_{\max} - \rho(a - x, t)$ for all $t \geq 0$.

Your solution goes here

Programming

Let $\rho_{\max} = 1$ and $u_{\max} = 1$. You will use your numerical method to compute a numerical solution to the PDE and plot the solution over the interval $-2\pi \leq x \leq 2\pi$ at selected times to provide a picture of what is going on with the solution at key times. The plots should be submitted, but the code will not be collected or graded.

Problem 2

Let $\rho(x, 0) = \frac{\rho_{\max}}{2}(1 + \sin(x))$ for the remaining parts of this problem. At $t = 0$, where will the cars move slowest? Plot the density profile at this time.

Your solution goes here

Problem 3

At $t = 0$, where do the traffic waves move slowest?

Your solution goes here

Problem 4

When will the first shocks form? Let this time be T.

Your solution goes here

Problem 5

Where will the first shocks form?

Your solution goes here

Problem 6

Plot the density profile at $t = T$ and $t = \frac{T}{2}$. What qualitative event happens to the density profile at $t = T$?

Your solution goes here

Problem 7

With what velocity will those shocks move?

Your solution goes here

Problem 8

For how long S will there be stopped cars?

Your solution goes here

Problem 9

Plot the density profile at $t = S$. What qualitative event happens to the density profile at $t = S$?

Your solution goes here

Problem 10

Plot the density profile at $t = 4T$ and $t \to \infty$. What is the long-term behavior of the density profile?

Your solution goes here