Math 142-2, Homework 7

Your name here

Problem 78.2

Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$ and that the initial traffic density is

$$\rho(x,0) = \begin{cases} \frac{\rho_{\max}}{5} & x < 0 \\ \frac{3\rho_{\max}}{5} & x > 0. \end{cases}$$

(a) Sketch the initial density.

Your solution goes here

(b) Determine and sketch the density at later times.

Your solution goes here

(c) Determine the path of a car (in space-time) which starts at $x = -x_0$ (behind $x = 0$).

Your solution goes here

(d) Determine the path of a car (in space-time) which starts at $x = x_0$ (ahead of $x = 0$).

Your solution goes here

Problem 78.3

Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$ and at $t = 0$, the traffic density is

$$\rho(x,0) = \begin{cases} \frac{\rho_{\max}}{3} & x < 0 \\ \frac{2\rho_{\max}}{3} & x > 0. \end{cases}$$

Why does the density not change in time?

Your solution goes here

Problem 78.4

Referring to the problem in Sec. 78, show algebraically that the value of the shock velocity is between the velocities of the two density waves.

Your solution goes here

Problem 79.2

Suppose that

$$\rho(x,0) = \begin{cases} \rho_0 & x > 0 \\ 0 & x < 0 \end{cases}$$

Determine the velocity of the shock. Briefly give a physical explanation of the result.

Your solution goes here

Problem 79.3

The initial traffic density on a road is

$$\rho(x,0) = \begin{cases} 0 & x \le 0 \\ \frac{\rho_{\max} x}{L} & 0 < x < L \\ \rho_{\max} & x \ge L \end{cases}$$

Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$.

(a) Sketch the initial density.

Your solution goes here

(b) Show that all characteristics from the interval $0 < x < L$ (and $t = 0$) intersect at the point $x = L/2$, $t = L/(2u_{\max})$.

Your solution goes here

(c) A traffic shock will form at this point. Find its subsequent motion.

Your solution goes here

(d) Sketch the $x - t$ plane, showing the shock and the characteristics necessary to determine $\rho(x,t)$.

Your solution goes here

(e) Sketch $\rho(x,t)$ before and after the shock.

Your solution goes here

(f) Describe briefly how the individual automobiles behave (do not determine their paths mathematically).

Your solution goes here

Problem 80.1

Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$.

(a) Show that the time of intersection of neighboring characteristics (corresponding to the collision of two observers initially at x_1 and x_2 moving with constant density ρ_1 and ρ_2) is

$$t = \frac{\rho_{\max}}{2u_{\max}\frac{\Delta\rho}{\Delta x}},$$

where $\Delta x = x_2 - x_1$ and $\Delta\rho = \rho_2 - \rho_1$.

Your solution goes here

(b) Extend this result to the limit as $x_2 \to x_1$ to determine when a shock will form from the characteristics that originate in the vicinity of some location x_1.

Your solution goes here

(c) If at $t = 0$,

$$\rho(x,0) = \rho_{\max}\exp\left(-\frac{x^2}{L^2}\right).$$

(1) Sketch the initial density.

Your solution goes here

(2) Determine the time of the first shock.

Your solution goes here

(3) Where does this shock first occur?

Your solution goes here

Problem 82.1

Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$. If the initial density is

$$\rho(x,0) = \begin{cases} \rho_1 & x < 0 \\ \rho_2 & a > x > 0 \\ \rho_3 & x > a \end{cases}$$

with $0 < \rho_1 < \rho_2 < \rho_3 < \rho_{\max}$, then determine the density at later times. [Hint: See exercise 77.1. Calculate the shock between ρ_1 and ρ_2. Show that this shock moves faster than the shock between ρ_2 and ρ_3. What happens after these two shocks meet?]

Your solution goes here

Problem 82.2

Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$ and that the initial traffic density is

$$\rho(x,0) = \begin{cases} \rho_1 & |x| > a \\ \rho_0 & |x| < a \end{cases}$$

where $\rho_1 > \rho_0$. Determine the density at later times.

Your solution goes here