Math 142-2, Homework 3

Your name here

Problem 18.4

Consider a spring-mass system with a nonlinear restoring force satisfying

$$m\frac{d^2x}{dt^2} = -kx - \alpha x^3,$$

where $\alpha > 0$ and $k > 0$. Which positions are equilibrium positions? Are they stable?

Your solution goes here

Problem 19.6

Suppose a mass m located at (x, y) is acted upon by a force field, $\mathbf{F}$ (i.e., $m(d^2\mathbf{x}/dt^2) = \mathbf{F}$). The kinetic energy is defined as

$$\frac{m}{2}\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right);$$

it again equals $\frac{1}{2}mv^2$. The potential energy is defined as

$$-\int_{\mathbf{x}_1}^{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s},$$

where $\mathbf{x}_1$ is any fixed position. If there exists a function $\phi(x, y)$ such that

$$\mathbf{F} = -\nabla\phi,$$

then show that the total energy (kinetic energy plus potential energy) is conserved. Such a force field $\mathbf{F}$ is called a *conservative force field*. In this case show that the potential energy equals $\phi(\mathbf{x}) - \phi(\mathbf{x}_1)$.

Your solution goes here

Problem 22.1

Suppose a spring-mass system is on a table retarded by a Coulomb frictional force (equation 10.3):

$$m\frac{d^2x}{dt^2} + kx = F_f, \qquad \text{where } F_f = \begin{cases} \gamma & \text{if } \frac{dx}{dt} < 0 \\ -\gamma & \text{if } \frac{dx}{dt} > 0. \end{cases}$$

(a) If $dx/dt > 0$, determine the energy equation. Sketch the resulting phase plane curves. [Hint: By completing the square show that the phase plane curves are ellipses centered at $v = 0$, $x = -\gamma/k$; not centered at $x = 0$.]

Your solution goes here

(b) If $dx/dt < 0$, repeat the calculation of part (a). [Hint: The ellipses now are centered at $v = 0$, $x = \gamma/k$.]

Your solution goes here

(c) Using the results of (a) and (b), sketch the solution in the phase plane. Show that the mass stops in a finite time!!!

Your solution goes here

(d) Consider a problem in which the mass is initially at $x = 0$ with velocity v_0. Determine how many times the mass passes $x = 0$ as a function of v_0.

Your solution goes here

Problem 23.1

Assume that a nonlinear pendulum is initially at its stable equilibrium position.

(a) How large an initial angular velocity is necessary for the pendulum to go completely around?

Your solution goes here

(b) At what initial angular velocity will the pendulum never pass its equilibrium position again?

Your solution goes here

Problem 24.2

Assume that the forces acting on a mass are such that the potential energy is the function of x shown in the figure in the book. Sketch the solutions in the phase plane. Describe the different kinds of motion that can occur.

Your solution goes here

Problem 26.2

Consider a linear oscillator with linear friction:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

(a) Show that $E = m/2(dx/dt)^2 + (k/2)x^2$ is a decreasing function of time.

Your solution goes here

(b) Let $v = dx/dt$ and show that $dv/dx = (-cv - kx)/(mv)$.

Your solution goes here

(c) Show that if $c^2 < 4mk$, then $v = \lambda x$ is not a solution in the phase plane.

Your solution goes here

(d) If $m = 1$, $c = 1$, $k = 1$, then roughly sketch the solution in the phase plane. (Use known information about the time-dependent solution to improve your sketch.)

Your solution goes here

(e) Show that if $c^2 > 4mk$, then $v = \lambda x$ is a solution in the phase plane for two different values of λ. Show that both values of λ are negative.

Your solution goes here

(f) If $m = 1$, $c = 3$, $k = 1$, then roughly sketch the solution in the phase plane. [Hint: Use the results of part (e)].

Your solution goes here

(g) Explain the qualitative differences between parts (d) and (f).

Your solution goes here

Problem 26.5

Briefly explain why only one solution curve goes through each point in the phase plane except for an equilibrium point in which case there may be more than one.

Your solution goes here