Math 142-2, Homework 7

Your name here

May 23, 2014

Problem 72.5

Sketch $dq/d\rho$ as a function of x, for fixed $t > 0$ (after the light turns green).

Your solution goes here

Problem 73.1

Assume that the traffic density is initially

$$\rho(x, 0) = \begin{cases} \rho_{\max} & x < 0 \\ \rho_{\max}/2 & 0 < x < a \\ 0 & a < x. \end{cases}$$

Sketch the initial density. Determine and sketch the density at all later times. Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$.

Your solution goes here

Problem 73.2

Calculate the maximum acceleration of a car which starts approximately one car length behind the traffic light (i.e., $x(0) = -1/\rho_{\max}$). Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$.

Your solution goes here

Problem 73.3

Calculate the velocity of a car at the moment it starts moving behind a light. Assume that $u = u_{\max}(1 - \rho/\rho_{\max})$.

Your solution goes here

Problem 73.6

Assume

$$u = u_{\max}\left(1 - \frac{\rho^2}{\rho_{\max}^2}\right).$$

Determine the traffic density that results after an infinite line of stopped traffic is started by a red traffic light turning green.

Your solution goes here

Problem 73.9

At what velocity does the information that the traffic light changed from red to green travel?

Your solution goes here

Problem 74.2

Assume $u(\rho) = u_{\max}(1 - \rho^2/\rho_{\max}^2)$ **and**

$$\rho(x,0) = \begin{cases} \rho_{\max} & x < 0 \\ \rho_{\max}(L-x)/L & 0 < x < L \\ 0 & L < x. \end{cases}$$

Determine $\rho(x,t)$**.**

Your solution goes here

Problem 74.3

Consider the following partial differential equation:

$$\frac{\partial \rho}{\partial t} - \rho^2 \frac{\partial \rho}{\partial x} = 0 \qquad -\infty < x < \infty.$$

(a) Why can't this equation model a traffic flow problem?

Your solution goes here

(b) Solve this partial differential equation by the method of characteristics, subject to the initial conditions:

$$\rho(x,0) = \begin{cases} 1 & x < 0 \\ 1-x & 0 < x < 1 \\ 0 & 1 < x. \end{cases}$$

Your solution goes here

Problem 74.4

Consider the example solved in this section. What traffic density should be approached as $L \to 0$? Verify that as $L \to 0$ equation 74.9 approaches the correct traffic density.

Your solution goes here

Additional Problem 1

Consider the model (in polar coordinates):

$$\frac{dr}{dt} = \sin(r) \qquad \frac{d\theta}{dt} = \cos(r).$$

(a) Identify all equilibria and identify their stability (stable, neutrally stable, or unstable) and shape (node, saddle, spiral/loop).

Your solution goes here

(b) Identify all limit cycles and identify them as stable or unstable.

Your solution goes here

(c) For each limit cycle identify whether the limit cycle trajectories are clockwise or counterclockwise.

Your solution goes here

(d) One of the tools we have used for sketching phase planes has been isoclines, or places where the slope is horizontal ($\frac{dy}{dt} = 0$) or vertical ($\frac{dx}{dt} = 0$). In this case, however, it is more convenient to consider places where the slope is radial ($\frac{d\theta}{dt} = 0$) or tangential ($\frac{dr}{dt} = 0$), along with arrows as before. Sketch the phase plane for this model (for $r \leq 2\pi$) by sketching these polar isoclines, the equilibria, and the limit cycles. Also sketch an estimate for what you think the trajectories might look like.

Your solution goes here

Additional Problem 2

Consider the model (for all x and y):

$$\frac{dx}{dt} = (x-y)(1-x^2-y^2) \qquad \frac{dy}{dt} = (x+y)(1-x^2-y^2).$$

(a) Identify all equilibria. Identify the stability (stable, neutrally stable, or unstable) and shape (node, saddle, spiral/loop) for the simple equilibrium (it should be clear which one this is). What can be said about the stability of the other equilibria? [Hint: there is more than one equilibrium for this model.]

Your solution goes here

(b) Identify all limit cycles and identify them as stable or unstable. [Hint: be careful.]

Your solution goes here

(c) Sketch a phase plane for this model, which should include all equilibria, limit cycles, and sample trajectories. (You may draw isoclines or other guides if this helps you with the trajectories.)

Your solution goes here

Additional Problem 3

Let $N(a,t)$ be the number of individuals with age a at time t, which is valid for $a \geq -g$ and $t \geq 0$, where g is the gestation time (time between conception and birth) is g. The conception rate $b(a)$ and death rate $c(a)$ are functions of age. The initial age distribution is $f(a)$. Derive a *continuous* model for the evolution of the population and its age distribution over time. Be sure to completely specify your model.

Your solution goes here