Math 142-2, Homework 3

Your name here

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Problem 44.3

Consider the following nonlinear systems:

(i) $\dfrac{dx}{dt} = e^x - 1 \qquad \dfrac{dy}{dt} = ye^x$

(ii) $\dfrac{dx}{dt} = x^2 + y^2 - 1 \qquad \dfrac{dy}{dt} = x + y$

(iii) $\dfrac{dx}{dt} = x^2 + y^2 - 5 \qquad \dfrac{dy}{dt} = x^2 + 2y^2 - 9$

(iv) $\dfrac{dx}{dt} = x^2 + y^2 - 1 \qquad \dfrac{dy}{dt} = x - 4$

(a) Determine all real equilibrium solutions.

(b) Linearize the nonlinear system in the vicinity of each equilibrium solution.

Your solution goes here

Problem 44.5

An equivalent method to linearize equation 44.1 in the neighborhood of an equilibrium population is to directly expand equation 44.1 via its Taylor series around the equilibrium population. Do this and show that equation 44.6 results.

Your solution goes here

Problem 45.1

Consider

$$\frac{dx}{dt} = ax + by$$

$$\frac{dy}{dt} = cx + dy,$$

for the following cases:

(a) $a = 0$, $b = 1$, $c = -4$, $d = 0$

Your solution goes here

(c) $a = 2,\ b = 1,\ c = 1,\ d = 2$

Your solution goes here

(n) $a = 1,\ b = 0,\ c = 0,\ d = -3$

Your solution goes here

(o) $a = 4,\ b = 2,\ c = 2,\ d = 1$

Your solution goes here

Problem 45.2

Consider

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & -9 & 5 \\ 1 & -10 & 7 \\ 1 & -17 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Find the general solution. [Hints: Show that the eigenvalues are 1, 2, and 3.]

Your solution goes here

Problem 46.1

Show that the solution $x = 0,\ y = 0$ of the linear system of equations 45.1 is stable if

$$a + d < 0 \text{ and } ad - bc > 0.$$

Your solution goes here

Problem 46.4

Consider

$$\frac{dx}{dt} = x(a - 4 + 2y) - a \qquad \frac{dy}{dt} = y(a + 1 - x) - 2a,$$

where a is a constant that may be positive or negative.

(a) Show that $x = 1,\ y = 2$ is an equilibrium solution.

Your solution goes here

(b) Linearize this system near this equilibrium. Show that

$$\frac{dx_1}{dt} = ax_1 + \beta y_1 \qquad \frac{dy_1}{dt} = -\beta x_1 + ay_1,$$

where x_1, y_1 are respective displacements from equilibrium. What is β?

Your solution goes here

(c) Given that the linearization is of the form above, for what values of a and β is $x = 1$, $y = 2$ stable? [Hint: Eliminate x_1 or y_1.]

Your solution goes here

Problem 47.1

Consider the systems

$$(1) \quad \frac{dx}{dt} = 2x \qquad \frac{dy}{dt} = x - 3y \qquad (2) \quad \frac{dx}{dt} = 4x \qquad \frac{dy}{dt} = 2x - 6y$$

Show that the phase plane analysis of both problems are the same although their solutions are entirely different.

Your solution goes here

Problem 49.1

Consider the following three-species ecosystem:

$$\frac{dF}{dt} = F(a - cS) \qquad \frac{dS}{dt} = S(-k + \lambda F - mG) \qquad \frac{dG}{dt} = G(-e + \sigma S)$$

Assume the coefficients are positive constants. Describe the role each species plays in this ecological system.

Your solution goes here

Problem 50.2

Consider the predator-prey model with $b \neq 0$, equation 50.1. Calculate all possible equilibrium solutions. Compare these populations to the ones which occur if $b = 0$. Briefly explain the qualitative and quantitative differences between the two cases, $b = 0$ and $b \neq 0$.

Your solution goes here