Ideal Gas Law

Lecture Notes (Math 142-1)

November 25, 2015

1 Temperature

- Defining temperature is very difficult
  - Thermodynamics
  - *Very* deep rabbit hole
  - Tip-toe around it
- Modeling assumptions instead
  - Adding energy (heat) increases temperature
  - Assume energy input proportional to temperature increase (empirical)
  - Assume energy input proportional to amount of gas
  - $\Delta KE = \hat{R}N\Delta T$
  - $KE = \hat{R}N(T - T_0)$
    - $T = T_0$ when $KE = 0$
    - T_0 observed to be the same for different types of gas (empirical)
    - T_0 is *absolute zero*
    - Measure temperature in Kelvin (K) so $T_0 = 0$
    - $KE = \hat{R}NT$
- $PV = \frac{2}{3}KE = \frac{2}{3}\hat{R}NT$

2 Ideal gas law

- $PV = kNT$
  - $k = \frac{2}{3}\hat{R}$
  - k observed to be the same for all gases (empirical)
  - Boltzmann constant: $k \approx 1.381 \times 10^{23}\, J\, K^{-1}$
- Measure amount of gas in *moles*
  - Moles: n
  - $N = N_A n$
  - Avogadro's number: $N_A \approx 6.022 \times 10^{23}\, mol^{-1}$
  - $1\, mol$ of gas is 6.022×10^{23} molecules of gas
  - Convenient macroscopic description

    * $1\,mol$ of hydrogen gas is $2.016\,g$
    * $1\,mol$ of helium gas is $4.003\,g$
    * $1\,mol$ of nitrogen gas is $28.01\,g$
    * $1\,mol$ of oxygen gas is $32.00\,g$
    * $1\,mol$ of carbon dioxide gas is $44.00\,g$
    * Nearly integers
  - Gas constant: $R = N_A k \approx 8.314\,J\,K^{-1}\,mol^{-1}$
  - $PV = nRT$

3 Internal energy

- Internal energy U
- KE is part of it
- Can be other contributions
  - Molecules can rotate
  - Molecular bonds can vibrate (like a spring getting longer and shorter)
- When adding energy (eg, heating the gas)
  - Part of energy goes into KE
  - Rest goes elsewhere
- Assume constant fraction α goes into KE, $0 < \alpha \leq 1$
- $KE = \alpha U$
- Note that KE contributes to pressure, but U as a whole does not
- Note that U is well-defined only up to a shift; $KE = \alpha U$ effectively fixes this

4 Equilibrium

- Molecules collide
- Exchange energy with each other
- Colliding off center can make molecule rotate
- Hitting atoms can make the bonds vibrate
- Hitting at an angle mixes up velocity components
- Everything gets *mixed up* over time
- Gas is normally close to equilibrium
- This is why gases appear isotropic
- Gas velocities end up in a characteristic velocity distribution