Math 142-1, Group work 8

Problem 1

Solve the PDE for $f(x,t)$ given the initial conditions $f(x,0) = s(x)$

$$\frac{\partial f}{\partial t}(x,t) + \frac{x}{t+1}\frac{\partial f}{\partial x}(x,t) = f(x,t) + t + x$$

Problem 2

Solve the PDE for $f(x,y,t)$ given the initial conditions $f(x,y,0) = s(x,y)$

$$\frac{\partial f}{\partial t}(x,y,t) + \frac{\partial f}{\partial x}(x,y,t) + \frac{\partial f}{\partial y}(x,y,t) = 1$$

Problem 3

Assume the car following model $\hat{u}(\rho) = u_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right)$, but to make things simple, let $u_{\max} = 1$ and $\rho_{\max} = 1$. Now, $\hat{u}(\rho) = 1 - \rho$. Investigate the qualitative behavior of this PDE with the ranges of time and space, initial conditions, and boundary conditions specified in the table below. You may use your program from project 2 (or the utility on the class website, or a friend's program from project 2) to help you with this. There are many aspects that you should familiarize yourself with: where and when do shocks form, how quickly and in what direction do they move, what happens when shocks meet, what happens when shocks meet rarefactions, and how do the density profiles change with time? Try to predict how you think the density profile will evolve; then check it numerically. With some practice, you *should* be able to fairly roughly draw the profile at later times without actually solving the PDE. This skill will be helpful to you on the final. (To numerically solve the problems with unbounded domain, you can just use a wide domain and approximate the fluxes by computing them from the initial densities at the endpoints; ignore what happens near the edges.)

$\rho(x,0)$	x range	t range	Flux F_L	Flux F_R
$\frac{1}{3}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\frac{1}{2}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\begin{cases} 0 & x<0 \\ 1 & x>0 \end{cases}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\begin{cases} 1 & x<0 \\ 0 & x>0 \end{cases}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\begin{cases} \frac{1}{6} & x<0 \\ \frac{2}{3} & x>0 \end{cases}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\begin{cases} \frac{5}{6} & x<0 \\ \frac{1}{3} & x>0 \end{cases}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\begin{cases} 0 & \lvert x\rvert<1 \\ 1 & \lvert x\rvert>1 \end{cases}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\begin{cases} 1 & \lvert x\rvert<1 \\ 0 & \lvert x\rvert>1 \end{cases}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\begin{cases} 0 & x<-1 \\ \frac{1}{3} & \lvert x\rvert<1 \\ 1 & x>1 \end{cases}$	$(-\infty,\infty)$	$[0,2]$	-	-
$\begin{cases} \frac{1}{6} & x<-1 \\ \frac{1}{3} & \lvert x\rvert<1 \\ 1 & x>1 \end{cases}$	$(-\infty,\infty)$	$[0,2]$	-	-
0	$[-1,1]$	$[0,2]$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{3}$	$[-1,1]$	$[0,2]$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$[-1,1]$	$[0,2]$	$\frac{1}{4}$	$\frac{1}{4}$
1	$[-1,1]$	$[0,2]$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}(1+\sin x)$	$[-\pi,\pi]$	$[0,4]$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}(1+\cos x)$	$[-\pi,\pi]$	$[0,4]$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}(1+\cos x)$	$[-\pi,\pi]$	$[0,4]$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}(3+\cos x)$	$[-\pi,\pi]$	$[0,4]$	$\frac{1}{4}$	$\frac{1}{4}$
e^{-x^2}	$(-\infty,\infty)$	$[0,4]$	-	-