Math 142-1, Group work 7

Problem 1

Suggest a traffic following model of the form $\dot{v}_k(t) = f(x_k, x_{k-1}, v_k, v_{k-1})$ that would lead to the relationship $\hat{u}(\rho) = u_{\max}\Big(1 - \frac{\rho}{\rho_{\max}}\Big)$.

Problem 2

Consider the model $\dot{v}_k(t) = -\lambda(x_k(t) - x_{k-1}(t) + d)$.

(a) What do λ and d represent?

(b) Why is $-\lambda$ more reasonable than λ?

(c) Show that no density-velocity relationship $\hat{u}(\rho)$ is implied by this model. (Hint: Show that $\hat{u}(\rho)$ implies $\dot{v}_k(t) = (v_k(t) - v_{k-1}(t))f(x_k(t) - x_{k-1}(t))$ for some function $f(x)$.)

(d) Do you think this model reasonably describes the behavior of drivers?

(e) Construct a traffic following model with a density-velocity relationship $\hat{u}(\rho)$ such that drivers try to maintain a fixed following time. (For example, drivers always try to stay two seconds behind the car in front.) Is this always possible?

(f) Is the model $\dot{v}_k(t) = \frac{v_k(t) - v_{k-1}(t)}{x_k(t) - x_{k-1}(t) + d}$ plausible?

Problem 3

Find the general solution to each of the differential equations below.

(a) $x''' = x$

(b) $tx' + ax = 0$

(c) $tx' + ax = b + ct$

(d) $t^2x'' + 4tx + 2x = 0$

(e) $\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} = 0$

(f) $\frac{\partial^2 f}{\partial x \partial y} = 0$

(g) $\left(\frac{\partial f}{\partial x}\right)^2 = \left(\frac{\partial f}{\partial y}\right)^2$

(h) $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y^2}$