# Math 142-1, Group work 4

## Problem 1

Five energy levels for a system are shown in the phase plane below. (a) List the energy levels (red, orange, green, blue, violet) in order from lowest energy to highest energy. (b) Mark all stable ("•") and unstable ("○") equilibria. (c) Sketch energy contours corresponding to all unstable equilibria (energy contours may contain more than one component; be sure to sketch them all). (d) Add arrows to all contours, including the ones you have added. (e) Sketch the potential energy function and show the energy levels corresponding to the five colored energy contours.
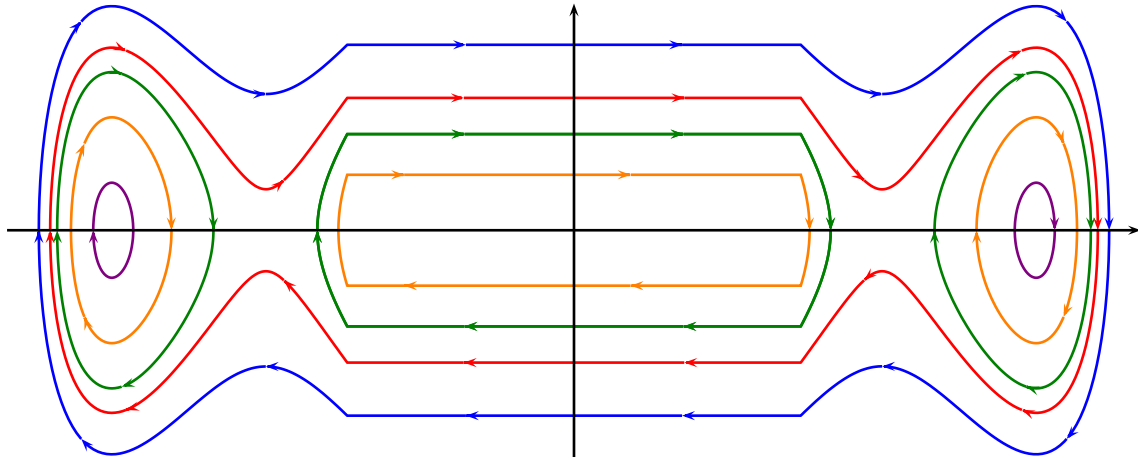


## Problem 2

A pulley of radius $r$ has wrapped around it a long cable with an object of mass $m$ hanging from it. The mass is at location $(r, x(t))$. The density of the pulley varies with the distance $z$ to the center of the pulley: $\rho(z) = cz$. The total mass of the pulley is $M$.

(a) Express $c$ in terms of $M$.

(b) What is the total potential energy of the system (in terms of $x(t)$)?

(c) What is the total kinetic energy of the system (in terms of $x(t)$ and $\dot{x}(t)$)?

(d) What is the total energy of the system?

(e) Find an ODE that describes the motion of the system.

(f) Find all equilibria and determine whether they are stable.