Math 135-2, Homework 5

Name: ______________________________ ID: ______________

Problem 69.1

Let (x_0, y_0) be an arbitrary point in the plane and consider the initial value problem

$$y' = y^2, \qquad y(x_0) = y_0.$$

Explain why Theorem A guarantees that this problem has a unique solution on some interval $|x - x_0| \leq h$. Since $f(x, y) = y^2$ and $\frac{\partial f}{\partial y} = 2y$ are continuous on the entire plane, it is tempting to conclude that this solution is valid for all x. By considering the solutions through the points $(0, 0)$ and $(0, 1)$, show that this conclusion is sometimes true and sometimes false, and that therefore the inference is not legitimate.

Problem 69.7

For what points (x_0, y_0) does Theorem A imply that the initial value problem

$$y' = y|y|, \qquad y(x_0) = y_0$$

has a unique solution on some interval $|x - x_0| \leq h$?

Problem 33.1

Find the Fourier series of the function defined by

$$\begin{cases} f(x) = \pi, & -\pi \leq x \leq \frac{\pi}{2}; \\ f(x) = 0, & \frac{\pi}{2} < x \leq \pi. \end{cases}$$

Problem 33.3

Find the Fourier series of the function defined by

$$\begin{cases} f(x) = 0, & -\pi \leq x < 0; \\ f(x) = \sin x, & 0 \leq x \leq \pi. \end{cases}$$

Problem 33.5

Find the Fourier series for the function defined by

(a) $f(x) = \pi$, $-\pi \le x \le \pi$
(d) $f(x) = \sin x$, $-\pi \le x \le \pi$
(c) $f(x) = \cos x$, $-\pi \le x \le \pi$
(d) $f(x) = \pi + \sin x + \cos x$, $-\pi \le x \le \pi$

Problem 35.1

Determine whether each of the following functions is even, odd, or neither:

$$x^5 \sin x, \quad x^2 \sin 2x, \quad e^x, \quad (\sin x)^3, \quad \sin x^2, \quad \cos(x + x^3), \quad x + x^2 + x^3, \quad \log \frac{1+x}{1-x}$$

Problem 36.2

Find the Fourier series for the functions defined by
(b) $f(x) = |x|, \qquad -2 \le x \le 2.$

Problem A

For the proof of the Picard theorem, we proved that any solution $\tilde{y}$ to the original ODE lies in the rectangle S. That is, if $x \in [x_0 - h, x_0 + h]$ then $\tilde{y}(x) \in [y_0 - Mh, y_0 + Mh]$. We used this to show that solutions to the ODE also must solve the integral equation. Adapt the proof to show that any solution $\bar{y}$ to the integral equation must lie in the same rectangle S. Recall that since $\bar{y}$ has not yet been established to be in S, we may not assume other parts of the proof apply to it (e.g., that it is also a solution to the ODE or that it is equal to the constructed solution y). Once this has been proven, full equivalence of the ODE and integral equations is established.