Math 135-2, Homework 3

Name: _____ ID: _____

Problem 53.4

Use the methods of both Examples 1 and 2 to solve each of the following differential equations:

(a) $y'' + 5y' + 6y = 5e^{3t}$, $y(0) = y'(0) = 0$.

Problem 53.8

The current $I(t)$ in an electric circuit with inductance L and resistance R is given by the equation (4) in Section 13:

$$L\frac{dI}{dt} + RI = E(t),$$

where $E(t)$ is the impressed electromotive force. If $I(0) = 0$, use the methods of this section to find $I(t)$ in each of the following cases:
(a) $E(t) = E_0 u(t)$
(b) $E(t) = E_0 \delta(t)$
(c) $E(t) = E_0 \sin \omega t$

Problem 69.2

Show that $f(x, y) = y^{1/2}$
(a) does not satisfy a Lipschitz condition on the rectangle $|x| \leq 1$ and $0 \leq y \leq 1$.
(b) does satisfy a Lipschitz condition on the rectangle $|x| \leq 1$ and $c \leq y \leq d$ where $0 < c < d$.

Problem 69.4

Show that $f(x, y) = xy^2$
(a) satisfies a Lipschitz condition on the rectangle $a \leq x \leq b$ and $c \leq y \leq d$.
(b) does not satisfy a Lipschitz condition on any strip $a \leq x \leq b$ and $-\infty \leq y \leq \infty$.

Problem A

The problem $yy' = 1$, $y(0) = 0$ seems like it should have no solution. Show that it actually has two solutions. How is this possible? This demonstrates that plugging the initial conditions into an ODE and producing a contradiction does not suffice to show that there is no solution.

Problem B

Consider the ODE $x^3y' = 2y$.
(a) Find all solutions if $y(0) = 0$.
(b) Find all solutions if $y(0) = 1$.

Problem C

Find the Lipschitz constant (or show that it does not have one) for each of the following functions on the indicated interval. (The Lipschitz constant is a *tight* bound for the Lipschitz condition.)
(a) $\cos x \sin x$, $(-\infty, \infty)$
(b) $|\sin x|$, $(-\infty, \infty)$

Problem D

Derive the time delay rule

$$L[u(x-a)f(x-a)] = e^{-ap}F(p).$$

For which choices a is this rule valid?