Math 135-2, Homework 2

Name: _____ ID: _____

Problem 50.2

Find the inverse Laplace transforms of

(c) $\dfrac{p+3}{p^2 + 2p + 5}$

Problem 50.3

Solve each of the following differential equations by the method of Laplace transforms:
(b) $y'' - 4y' + 4y = 0$, $y(0) = 0$ and $y'(0) = 3$
(d) $y'' + y' = 3x^2$, $y(0) = 0$ and $y'(0) = 1$

Problem 51.1

Show that

$$L[x \cos ax] = \frac{p^2 - a^2}{(p^2 + a^2)^2}$$

and use this result to find

$$L^{-1}\left[\frac{1}{(p^2 + a^2)^2}\right]$$

Problem 51.3

Solve each of the following differential equations:
(a) $xy'' + (3x - 1)y' - (4x + 9)y = 0$, $y(0) = 0$.

Problem 51.7

If $x > 0$, show formally that
(b) $f(x) = \displaystyle\int_0^\infty \frac{\cos xt}{1 + t^2}\, dt = \frac{\pi}{2} e^{-x}$

Problem 52.2

Solve each of the following integral equations:

(b) $y(x) = e^x \left[1 + \int_0^x e^{-t} y(t) \, dt \right]$

Problem 52.5

Show that the differential equation

$$y'' + a^2 y = f(x), \, y(0) = y'(0) = 0$$

has

$$y(x) = \frac{1}{a} \int_0^x f(t) \sin a(x - t) \, dt$$

as its solution.

Problem 53.2

Find the convolution of each of the following pairs of functions:
(a) $1, \, \sin at$
(c) $t, \, e^{at}$