Math 135-2, Homework 1

Name: ______________________________ ID: ______________

Problem 17.1

Find the general solution of each of the following equations:
(p) $16y'' - 8y' + y = 0$
(q) $y'' + 4y' + 5y = 0$
(r) $y'' + 4y' - 5y = 0$

Problem 17.5

The equation

$$x^2y'' + pxy' + qy = 0,$$

where p and q are constants, is called *Euler's equidimensional equation.* Show that the change of independent variable given by $x = e^z$ transforms it into an equation with constant coefficients, and apply this technique to find the general solution of each of the following equations:

(a) $x^2y'' + 3xy' + 10y = 0$

Problem 18.3

If $y_1(x)$ and $y_2(x)$ are solutions of

$$y'' + P(x)y' + Q(x)y = R_1(x)$$

and

$$y'' + P(x)y' + Q(x)y = R_2(x),$$

show that $y(x) = y_1(x) + y_2(x)$ is a solution of

$$y'' + P(x)y' + Q(x)y = R_1(x) + R_2(x).$$

This is called the principle of *superposition.* Use this principle to find the general solution of
(b) $y'' + 9y = 2\sin 3x + 4\sin x - 26e^{-2x} + 27x^3$

Problem 48.1

Evaluate the integrals in (8), (9), (11), (12), and (13).

Problem 48.2

Without integrating, show that
(a) $L[\sinh ax] = \dfrac{a}{p^2 - a^2},\ p > |a|$

Problem 48.4

Use the formulas given in the text to find the transform of each of the following functions:
(d) $4 \sin x \cos x + 2e^{-x}$

Problem 48.5

Find a function $f(x)$ whose transform is
(e) $\dfrac{1}{p^4 + p^2}$

Problem 49.2

In each of the following cases, graph the function and find its Laplace transform:
(a) $f(x) = u(x - a)$ **where** a **is a positive number and** $u(x)$ **is the unit step function defined by**

$$u(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

(b) $f(x) = [x]$ **where** $[x]$ **denotes the greatest integer** $\leq x$
(c) $f(x) = x - [x]$
(d) $f(x) = \begin{cases} \sin x & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } x > \pi \end{cases}$

Problem 49.4

Show explicitly that $L[x^{-1}]$ **does not exist.**

Problem 50.1

Find the Laplace transforms of
(b) $(1 - x^2)e^{-x}$