CS 230, Quiz 3

Solutions

You will have 10 minutes to complete this quiz. No books, notes, or other aids are permitted.

Problem 1 (3 points)

Clip the segment (in homogeneous coordinates) with endpoints $A = (-4, 1, 3, 5)$ and $B = (-2, 5, 3, 3)$.

Note that both vertices satisfy all of the clipping constraints except $y \leq w$, which B fails. Thus, we only need to clip against the plane $y = w$. Let

$$\begin{aligned}
P &= (1-\gamma)A + \gamma B \\
0 &= P_y - P_w \\
&= ((1-\gamma)A_y + \gamma B_y) - ((1-\gamma)A_w + \gamma B_w) \\
&= (1 - \gamma + 5\gamma) - (5(1-\gamma) + 3\gamma) \\
&= -4 + 6\gamma \\
\gamma &= \frac{2}{3} \\
P &= \frac{1}{3}A + \frac{2}{3}B \\
P &= \begin{pmatrix} -\frac{8}{3} \\ \frac{11}{3} \\ 3 \\ \frac{11}{3} \end{pmatrix}
\end{aligned}$$

Then, AP is the clipped segment.

Problem 2 (6 points)

Construct a 3×3 matrix that performs each of the following 2D operations (in homogeneous coordinates). If no such matrix exists, explain why.

(a) $(x, y) \rightarrow (y, x)$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) $(x, y) \to (1 + \frac{1}{x}, \frac{y}{x})$

$$(x, y, 1) \to (1 + x, y, x)$$
$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

(c) $(x, y) \to (\frac{1}{x}, \frac{1}{y})$

Clearing the fractions using homogeneous coordinates gives (y, x, xy), but xy cannot be achieved with a matrix. No such matrix exists.

(d) $(x, y) \to (\frac{y+1}{x+1}, 2)$

$$(x, y, 1) \to (y + 1, 2x + 2, x + 1)$$
$$\begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

(e) $(x, y) \to (\frac{x}{x+y}, \frac{y}{x+y})$

$$(x, y, 1) \to (x, y, x + y)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

(f) $(x, y) \to (\frac{ax+by+c}{rx+sy+t}, \frac{dx+ey+f}{rx+sy+t})$, for given constants $a, b, c, d, e, f, r, s, t$.

$$(x, y, 1) \to (ax + by + c, dx + ey + f, rx + sy + tx, y, x + y)$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ r & s & t \end{pmatrix}$$