Line Rasterization

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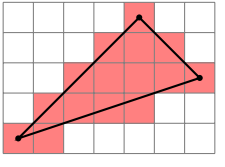
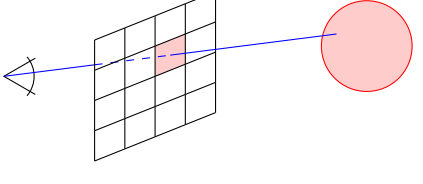
Raster Image

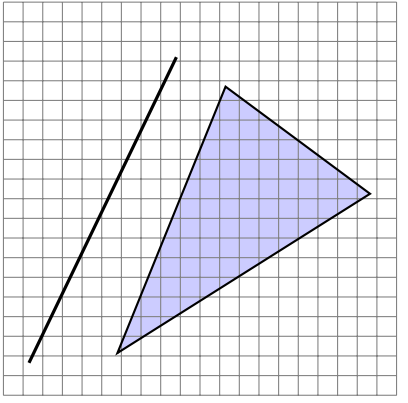
- Object oriented
  - for each object...



- Image oriented
  - for each pixel...



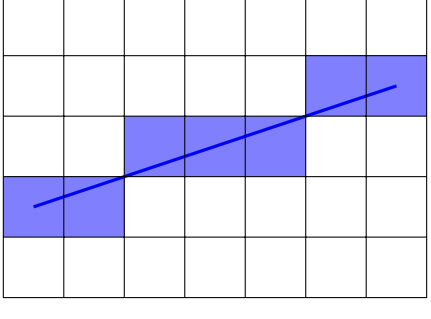
What is rasterization?



Rasterization is the process of determining which pixels are “covered” by the primitive

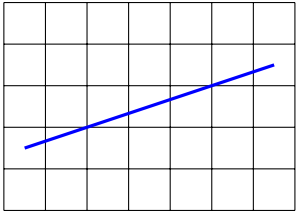
Rasterization

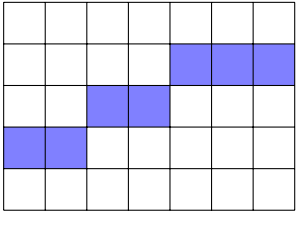
- In: 2D primitives (floating point)
- Out: covered pixels (integer)
- Must be fast (called **many times**)
- Visually pleasing
  - lines have constant width
  - lines have no gaps

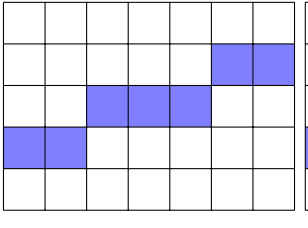


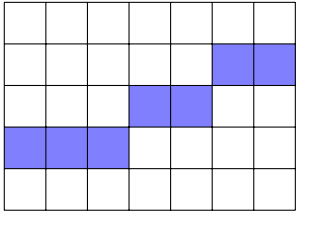
Rasterization choices

- Thin, no gaps
- Still have choices





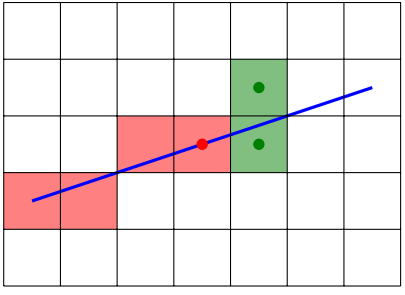




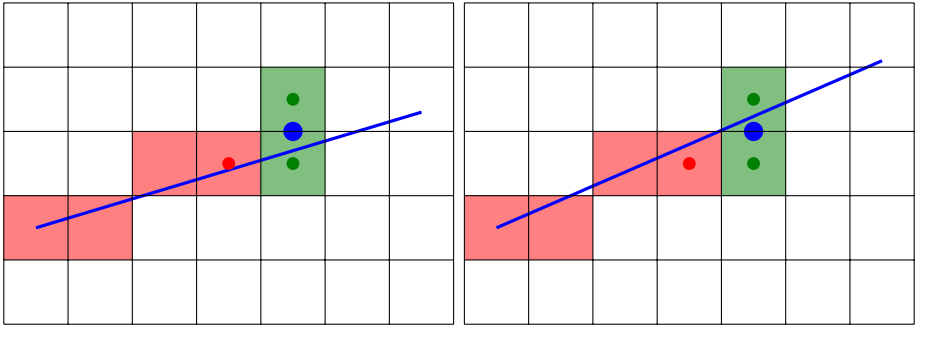
Midpoint algorithm

- Assume $0 \leq m \leq 1$
- Move from left to right
- Choose between $(x+1, y)$ and $(x+1, y+1)$

$y = y_0$
for $x = x_0, \ldots, x_1$ **do**
    draw(x, y)
    if ⟨condition⟩ **then**
        $y \leftarrow y + 1$



Check midpoint location



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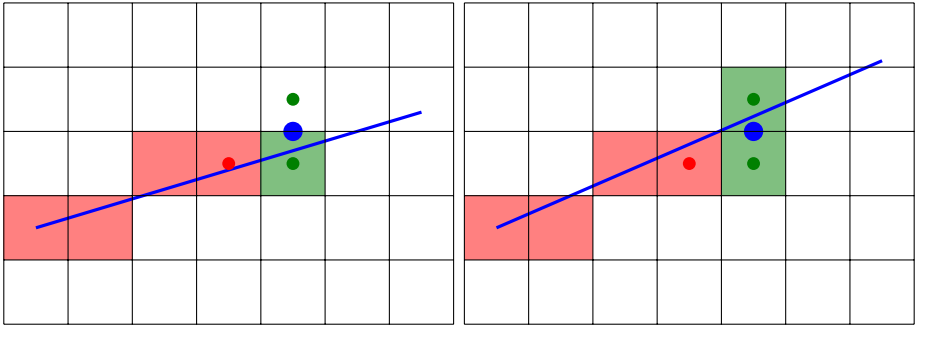

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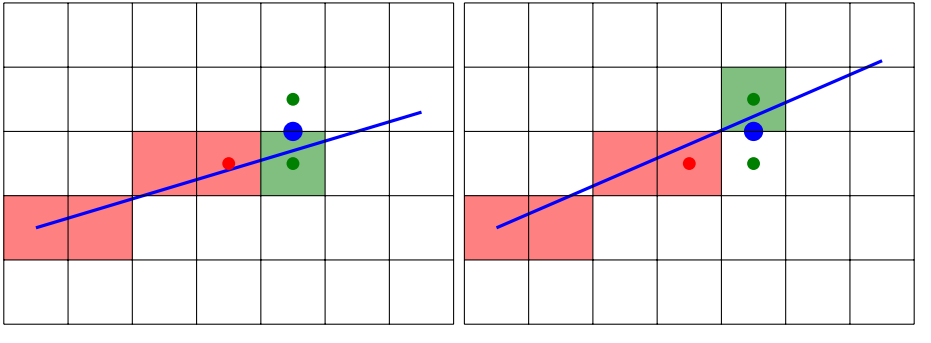
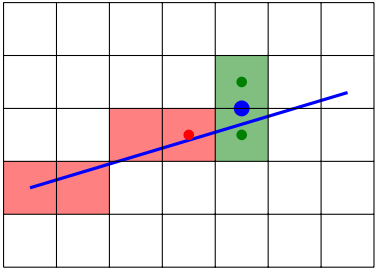


Criterion

Implicit line equation:

$$f(\mathbf{x}) = \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_0) = 0$$



Criterion

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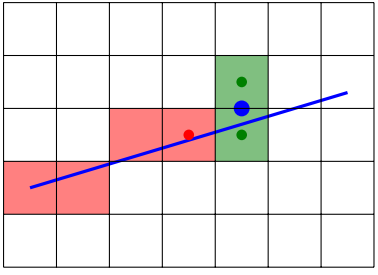
Evaluate f at midpoint:

$$f\left(x + 1, y + \frac{1}{2}\right) \overset{?}{<} 0$$



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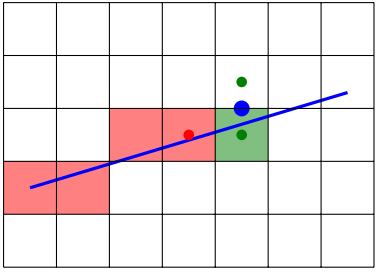
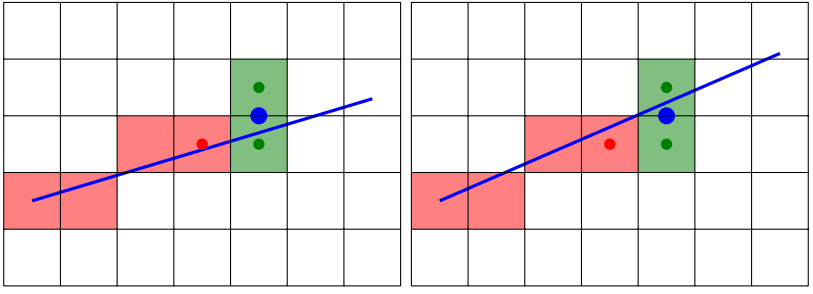


Midpoint algorithm ($0 \leq m \leq 1$)

$y \leftarrow y_0$
for $x = x_0, \ldots, x_1$ **do**
    draw(x, y)
    if $f(x+1, y+\frac{1}{2}) < 0$ **then**
        $y \leftarrow y+1$



Efficiency: incremental update

- Compute initial $f(x, y)$
- Compute next by updating previous
- Update with *one* addition

$$f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + (x_0 y_1 - x_1 y_0)$$

Efficiency: incremental update

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$$f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + (x_0 y_1 - x_1 y_0)$$
$$f(x + 1, y) = f(x, y) + (y_0 - y_1)$$

Efficiency: incremental update

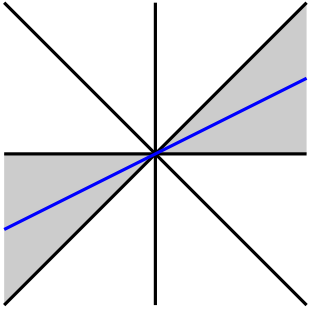
- Compute initial $f(x, y)$
- Compute next by updating previous
- Update with *one* addition

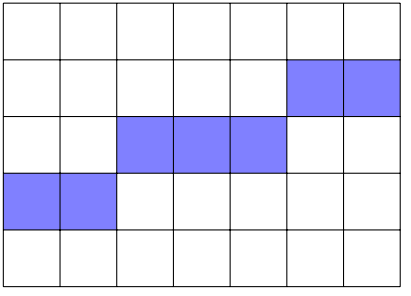
$$
\begin{aligned}
f(x, y) &= (y_0 - y_1)x + (x_1 - x_0)y + (x_0 y_1 - x_1 y_0) \\
f(x+1, y) &= f(x, y) + (y_0 - y_1) \\
f(x+1, y+1) &= f(x, y) + (y_0 - y_1) + (x_1 - x_0)
\end{aligned}
$$

Efficiency: incremental update

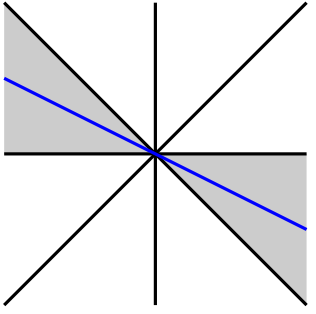
$y \leftarrow y_0$
$d \leftarrow f(x_0 + 1, y_0 + \frac{1}{2})$
for $x = x_0, \ldots, x_1$ **do**
    draw(x, y)
    if $d < 0$ **then**
        $y \leftarrow y + 1$
        $d \leftarrow d + (y_0 - y_1) + (x_1 - x_0)$
    else
        $d \leftarrow d + (y_0 - y_1)$

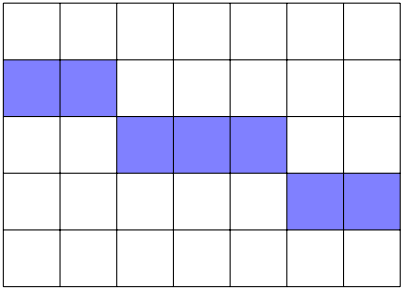
Other cases: $0 \leq m \leq 1$





Other cases: $-1 \leq m \leq 0$





Other cases: $|m| > 1$