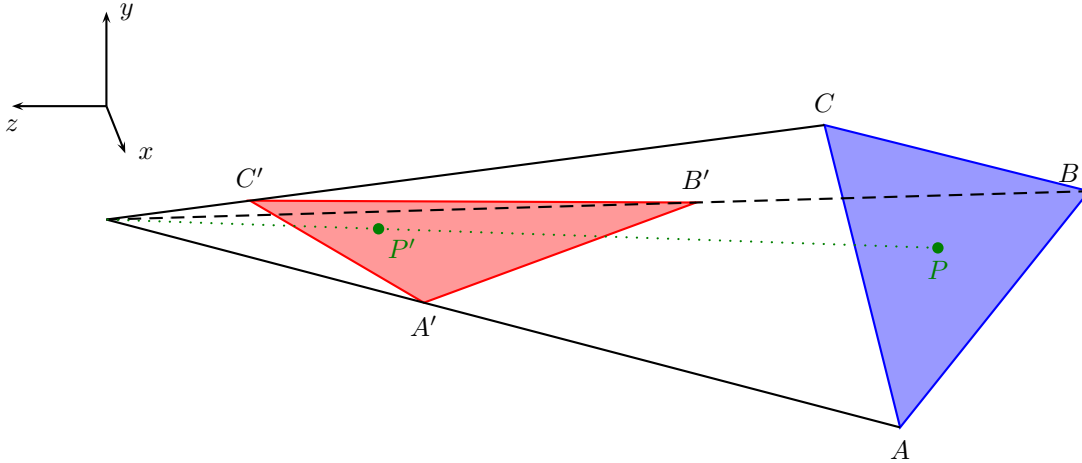


# Perspective Correct Interpolation

CS 130

1. Viewing frustum in camera space. Camera is at the origin.



2. Transform from  $A, B, C, P$  to  $A', B', C', P'$  by homogeneous matrix  $\mathbf{M}$ .

$$\begin{pmatrix} A'w_a \\ w_a \end{pmatrix} = \mathbf{M} \begin{pmatrix} A \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} B'w_b \\ w_b \end{pmatrix} = \mathbf{M} \begin{pmatrix} B \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} C'w_c \\ w_c \end{pmatrix} = \mathbf{M} \begin{pmatrix} C \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} P'w_p \\ w_p \end{pmatrix} = \mathbf{M} \begin{pmatrix} P \\ 1 \end{pmatrix}$$

3. The real barycentric weights are  $\alpha, \beta, \gamma$ . Because of the projection, they appear to be  $\alpha', \beta', \gamma'$ .

$$P = \alpha A + \beta B + \gamma C$$
$$P' = \alpha' A' + \beta' B' + \gamma' C'$$

4. While rasterizing, we can compute  $\alpha', \beta', \gamma'$  directly, but we will need the real weights  $\alpha, \beta, \gamma$  to correctly interpolate color.

5. Noting  $\alpha + \beta + \gamma = 1$ ,

$$\begin{aligned} \begin{pmatrix} P \\ 1 \end{pmatrix} &= \alpha \begin{pmatrix} A \\ 1 \end{pmatrix} + \beta \begin{pmatrix} B \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} C \\ 1 \end{pmatrix} \\ \mathbf{M} \begin{pmatrix} P \\ 1 \end{pmatrix} &= \alpha \mathbf{M} \begin{pmatrix} A \\ 1 \end{pmatrix} + \beta \mathbf{M} \begin{pmatrix} B \\ 1 \end{pmatrix} + \gamma \mathbf{M} \begin{pmatrix} C \\ 1 \end{pmatrix} \\ P'w_p &= \alpha A'w_a + \beta B'w_b + \gamma C'w_c \\ w_p &= \alpha w_a + \beta w_b + \gamma w_c \\ P' &= \frac{\alpha A'w_a + \beta B'w_b + \gamma C'w_c}{\alpha w_a + \beta w_b + \gamma w_c} \\ P' &= \frac{\alpha w_a}{\alpha w_a + \beta w_b + \gamma w_c} A' + \frac{\beta w_b}{\alpha w_a + \beta w_b + \gamma w_c} B' + \frac{\gamma w_c}{\alpha w_a + \beta w_b + \gamma w_c} C' \\ \alpha' &= \frac{\alpha w_a}{\alpha w_a + \beta w_b + \gamma w_c} \\ \beta' &= \frac{\beta w_b}{\alpha w_a + \beta w_b + \gamma w_c} \\ \gamma' &= \frac{\gamma w_c}{\alpha w_a + \beta w_b + \gamma w_c} \end{aligned}$$

6. This is the wrong way around. We have  $\alpha'$  but need  $\alpha$ .

$$\begin{aligned} k &= \frac{1}{\alpha w_a + \beta w_b + \gamma w_c} \\ \alpha' &= \alpha w_a k \\ \beta' &= \beta w_b k \\ \gamma' &= \gamma w_c k \\ \alpha &= \frac{\alpha'}{w_a k} \\ \beta &= \frac{\beta'}{w_b k} \\ \gamma &= \frac{\gamma'}{w_c k} \\ 1 &= \alpha + \beta + \gamma = \frac{\alpha'}{w_a k} + \frac{\beta'}{w_b k} + \frac{\gamma'}{w_c k} \\ k &= \frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c} \\ \alpha &= \frac{\frac{\alpha'}{w_a}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}} \\ \beta &= \frac{\frac{\beta'}{w_b}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}} \\ \gamma &= \frac{\frac{\gamma'}{w_c}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}} \end{aligned}$$

7. Can now use  $\alpha, \beta, \gamma$  to interpolate colors.