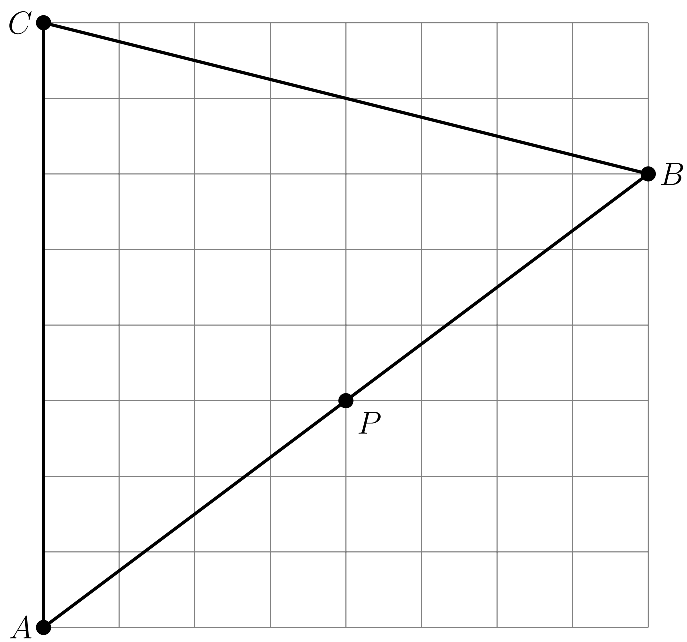
CS 130, Midterm

Solutions

Read the entire exam before beginning. **Manage your time carefully.** This exam has 34 points; you need 30 to get full credit. Additional points are extra credit. 34 points → 2.3 min/point. 30 points → 2.7 min/point.

Problem 1 (2 points)

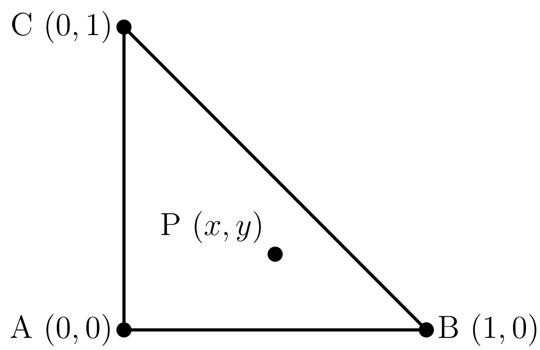
Compute the barycentric weights of the point P.



P is at the midpoint of one of the edges, which makes the barycentric weights easy: $\alpha = \beta = \frac{1}{2}$, $\gamma = 0$.

Problem 2 (2 points)

Find the barycentric weights for the point P in the triangle below.



To do this, we need to compute the areas of the triangles, which we can then use to compute barycentric weights. Note that all but one of the triangles we need have an edge that is horizontal or vertical, so $A = \frac{1}{2}bh$ is easy to compute.

$$\text{area}(ABC) = \frac{1}{2} \qquad \text{area}(APC) = \frac{x}{2} \qquad \text{area}(ABP) = \frac{y}{2}$$

$$\text{area}(PBC) = \text{area}(ABC) - \text{area}(APC) - \text{area}(ABP) = \frac{1-x-y}{2}$$

$$\alpha = \frac{\text{area}(PBC)}{\text{area}(ABC)} = 1 - x - y \qquad \beta = \frac{\text{area}(APC)}{\text{area}(ABC)} = x \qquad \gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)} = y$$

Problem 3 (2 points)

A raytracing scene has a yellow-colored sphere illuminated by a magenta-colored light. What color will the sphere appear in the image?

The sphere is color $(1,1,0)$ and the light is color $(1,0,1)$. These two vectors will be multiplied componentwise (whether ambient, specular, or diffuse), resulting in $(1,0,0)$, which is red. Depending on the lighting, this may be scaled down, resulting in a darker red. Note that a lighter red or pink is not possible.

Problem 4 (2 points)

Give reasonable RGB values for (1) blue, (2) dark blue, and (3) light blue.

Blue is $(0,0,1)$. Dark blue is a mixture of blue with black: $(0,0,0.5)$. Light blue is a mixture with white: $(0.5,0.5,1)$.

Problem 5 (2 points)

When viewed from 20 cm away, the whiteboard appears white. When viewed from a distance of 20 m away, the whiteboard appears to be approximately the same color and brightness. In the second case, the observer is standing 100 times farther away. Why does the whiteboard not appear 10,000 times dimmer (essentially completely black)?

In any fixed area of your retina, the light reaching your eye from any patch of the board will indeed be reduced by a factor of 10,000. However, the area of the board that reaches that part of your retina is multiplied by a factor of 10,000. The result is that the brightness stays about the same. You do still receive 10,000 times less light since the board shines that light over an area of your retina 10,000 times smaller.

Problem 6 (3 points)

Given two vectors $\vec{u}$ and $\vec{w}$, how do we determine whether the vectors are parallel? You may assume 3D.

$\vec{u} \times \vec{w} = \vec{0}$. Another solution would be to check, for example, $\frac{\vec{u}}{\|\vec{u}\|} = \frac{\vec{w}}{\|\vec{w}\|}$. $\vec{u} \cdot \vec{w} = \|\vec{u}\|\|\vec{w}\|$ is another solution.

Problem 7 (5 points)

Compute the normal direction for the implicit surface $f(x, y) = 2x^2y^2 - 5xy^2 + 2$ at the point $(2, -1)$. You do not need to simplify your answer.

$$\begin{aligned}
\mathbf{u} = \nabla f = \begin{pmatrix} 4xy^2 - 5y^2 \\ 4x^2y - 10xy \end{pmatrix} &= \begin{pmatrix} 4(2)(-1)^2 - 5(-1)^2 \\ 4(2)^2(-1) - 10(2)(-1) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \\
\|\mathbf{u}\| &= 5 \\
\mathbf{n} = \frac{\mathbf{u}}{\|\mathbf{u}\|} &= \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}
\end{aligned}$$

Problem 8 (2 points)

The equation $2z = x^2 + y^2$ represents a paraboloid. Compute *all* of the intersection *locations* between this paraboloid and the ray with endpoint $\mathbf{e} = \langle 0, -1, 3\rangle$ and direction $\mathbf{u} = \langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\rangle$.

$$\begin{aligned} x &= \frac{t}{3} \\ y &= \frac{-3-2t}{3} \\ z &= \frac{9+2t}{3} \\ 2z &= x^2 + y^2 \\ 2\left(\frac{9+2t}{3}\right) &= \left(\frac{t}{3}\right)^2 + \left(\frac{-3-2t}{3}\right)^2 \\ 6(9+2t) &= t^2 + (3+2t)^2 \\ 0 &= 5t^2 - 45 \\ t^2 &= 9 \\ t &= \pm 3 \end{aligned}$$

The negative solution does not lie on the ray, so $t = 3$. Plugging this in we get the intersection location $(1, -3, 5)$.

Problem 9 (2 points)

The equation $2z = x^2 + y^2$ represents a paraboloid. Given a point (x, y, z) that is on the paraboloid, compute the normal direction at that location.

This can be done easily as a parametric equation: $(u, v, (x^2 + y^2)/2)$. It can also be done as an implicit equation: $f(x, y, z) = x^2 + y^2 - 2z = 0$. The latter approach is simpler, so we will do it that way.

$$\begin{aligned} \nabla f &= \begin{pmatrix} 2x \\ 2y \\ -2 \end{pmatrix} \\ \mathbf{n} = \frac{\nabla f}{\|\nabla f\|} &= \frac{1}{\sqrt{x^2 + y^2 + 1}} \begin{pmatrix} x \\ y \\ -1 \end{pmatrix} \end{aligned}$$

Problem 10 (2 points)

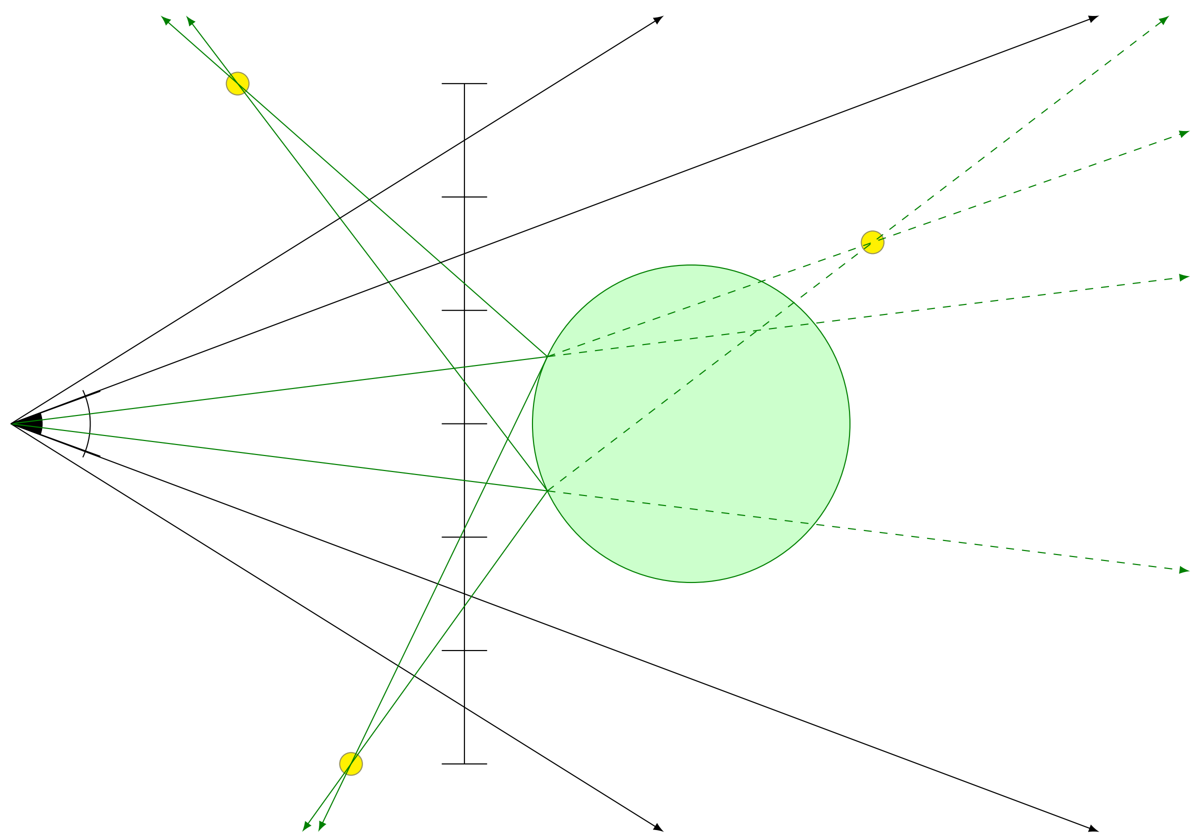
Let $\mathbf{z} = \mathbf{e} + t\mathbf{u}$, where $\mathbf{z}, \mathbf{e}, \mathbf{u}$ are vectors and are given to you. You may assume that $\mathbf{u}$ has been correctly normalized. Find t. *Be careful.*

Take the dot product by $\mathbf{u}$ and solve, which yields $t = (\mathbf{z} - \mathbf{e}) \cdot \mathbf{u}$.

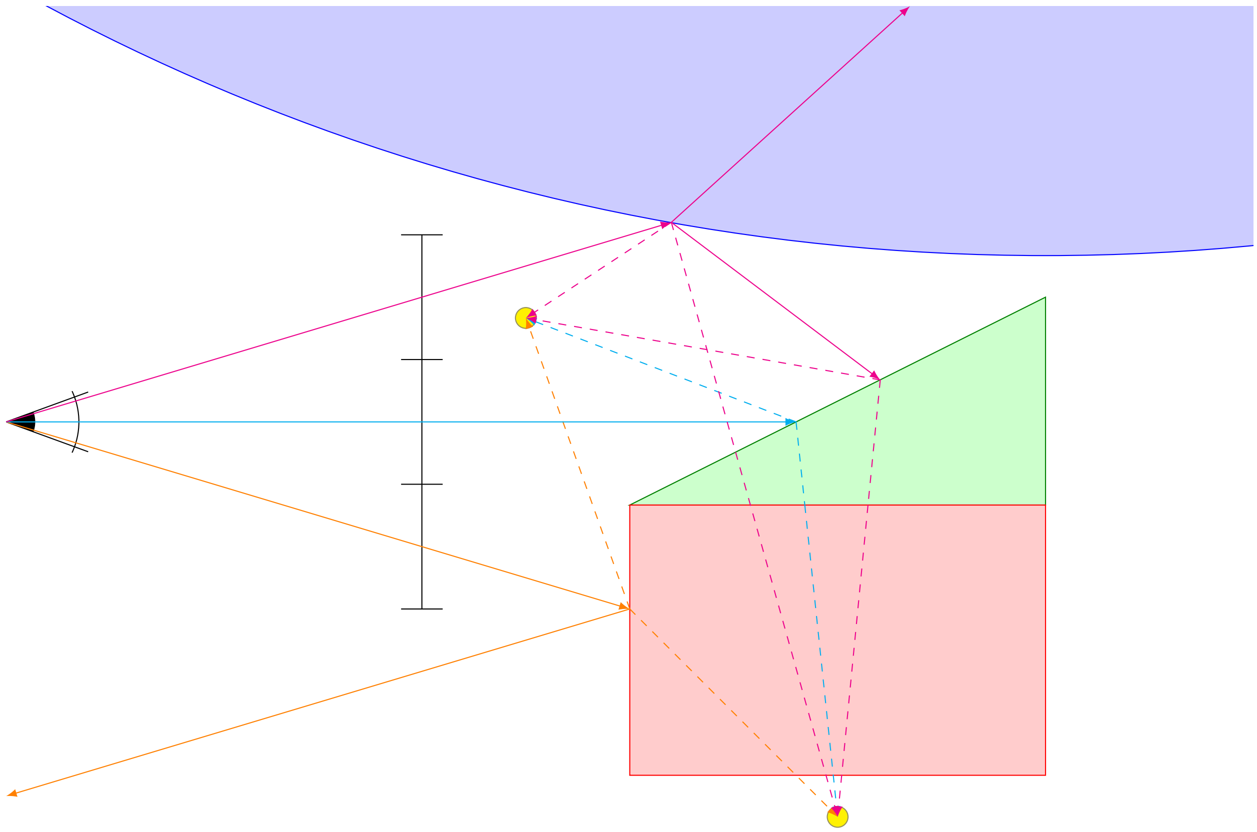
In the raytracing problems below, green objects are wood, red objects are reflective, and blue objects are transparent. The scenes are in 2D with a 1D image. yellow circles are point lights; the ray tracer supports shadows. Draw all of the rays that would be cast while raytracing each scene. Use a maximum recursion depth of 6. (Don't worry about precisely what counts as depth 6; I just care that recursion is being performed correctly when necessary and that important rays are not missing. There are no more than 25 rays in the "exact" solution.)

Problem 11 (4 points)

Note that this image has six pixels.



Problem 12 (4 points)



Problem 13 (2 points)

Why does a bump-mapped sphere cast a shadow that does not look bumpy?

Bump mapping only changes the color computation of the shader, not the physical geometry of the sphere. Since the sphere is geometrically smooth, so too are shadows and the silhouette. The intersection routines are not aware of the bump map.

[1]

[1]Total points: 34