CS 130, Final

Solutions

Read the entire exam before beginning. **Manage your time carefully.** This exam has 38 points; you need 34 to get full credit. Additional points are extra credit. 38 points $\to$ 3.0 min/point. 34 points $\to$ 3.5 min/point.

Problem 1 (2 points)

When working through 3D rotations, we defined a matrix $\mathbf{u}^*$ from a vector $\mathbf{u}$ such that matrix-vector multiplication was equivalent to cross product: $(\mathbf{u}^*)\mathbf{v} = \mathbf{u} \times \mathbf{v}$. Construct the matrix $\mathbf{u}^*$ explicitly from the components of $\mathbf{u} = \langle x, y, z \rangle$.

$$\mathbf{u}^* = \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$$

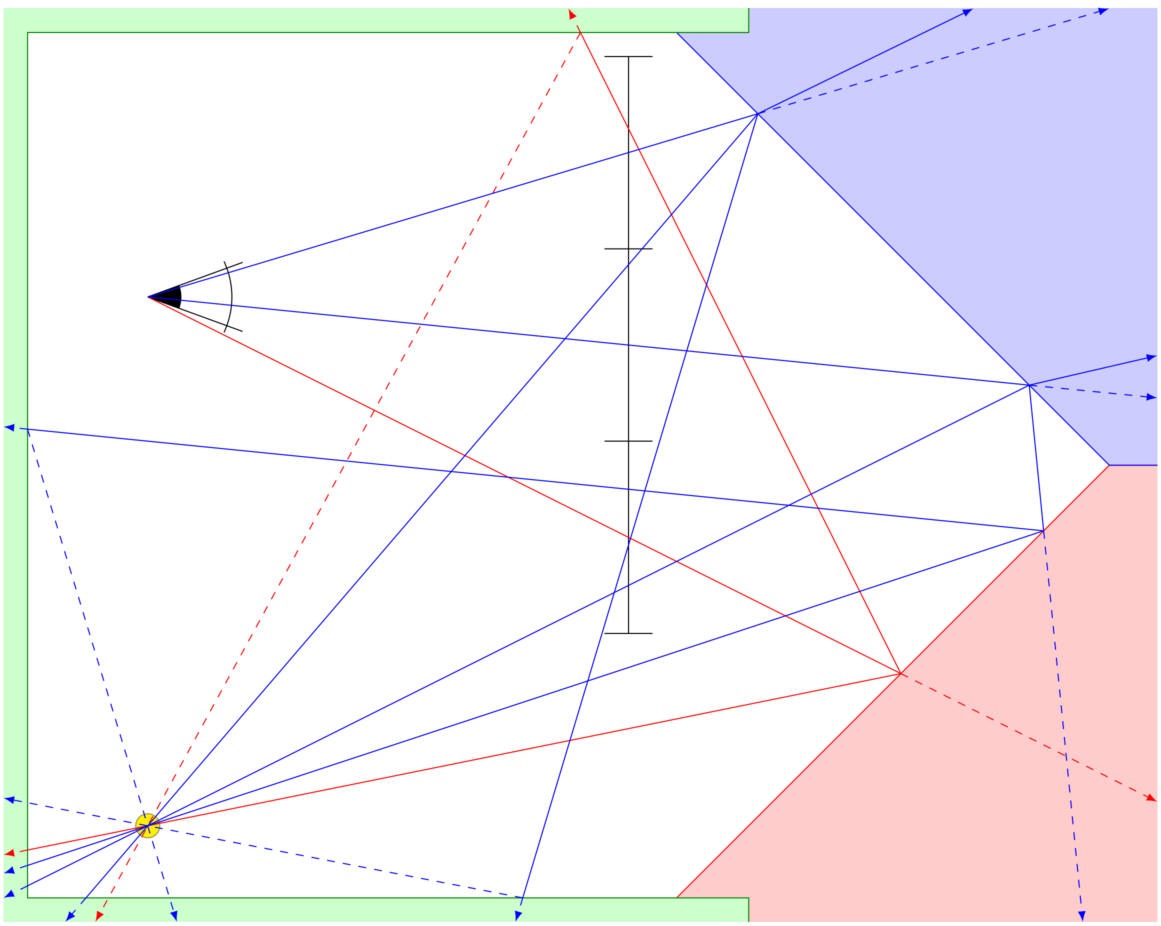
Problem 2 (2 points)

Given two vectors $\vec{u}$ and $\vec{w}$, construct a vector $\vec{q}$ orthogonal to both of them.

$\vec{q} = \vec{u} \times \vec{w}$

In the raytracing problems below, green objects are wood, red objects are reflective, and blue objects are transparent. The scenes are in 2D with a 1D image. yellow circles are point lights; the ray tracer supports shadows. Draw all of the rays that would be cast while raytracing each scene. Use a maximum recursion depth of 3. (Don't worry about precisely what counts as depth 3; I just care that recursion is being performed correctly when necessary and that important rays are not missing. There are no more than 30 rays in the "exact" solution.)

Problem 3 (5 points)



Problem 4 (2 points)

Let $\mathbf{z} = \mathbf{e} + t\mathbf{u}$, where $\mathbf{z}, \mathbf{e}, \mathbf{u}$ are vectors and are given to you. You may assume that $\mathbf{u}$ has been correctly normalized. Find t. *Be careful.*

Take the dot product by $\mathbf{u}$ and solve, which yields $t = (\mathbf{z} - \mathbf{e}) \cdot \mathbf{u}$.

Problem 5 (2 points)

What sort of activity might one perform in a fragment shader? Be specific.

This is normally where the color of a pixel is computed (eg, a phong shader). Texture mapping is also applied here.

Problem 6 (2 points)

Why do we use red, green, and blue when producing images? Why not two or four colors instead of three? Why not a different set of three colors?

The human eye uses three types of color-sensitive cells to distinguish colors; these cells are most sensitive to red, green, and blue light. The eye thus perceives other colors as combinations of these colors.

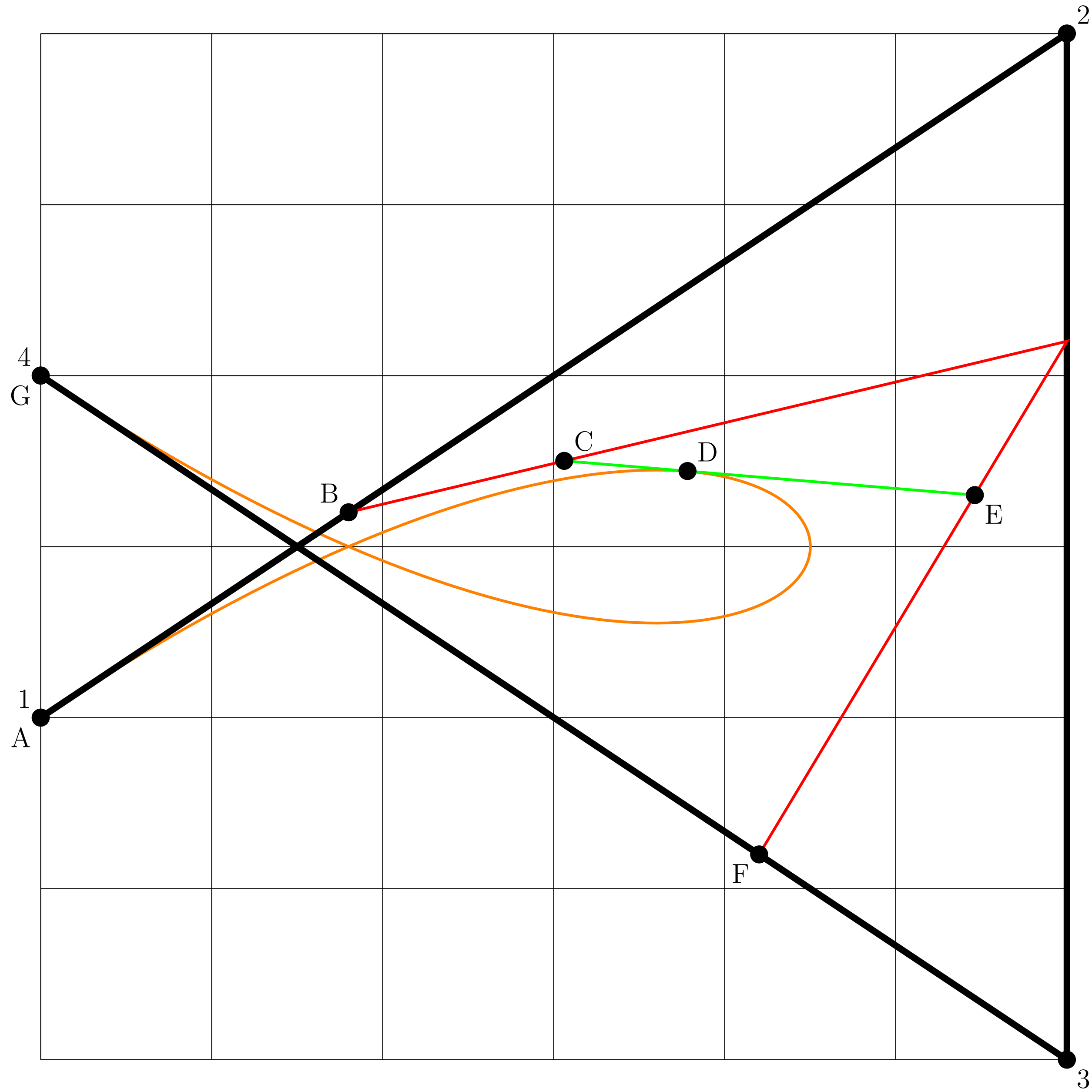
Problem 7 (2 points)

When working through 3D rotations, we defined a matrix $\mathbf{u}^*$ from a vector $\mathbf{u}$ such that matrix-vector multiplication was equivalent to cross product: $(\mathbf{u}^*)\mathbf{v} = \mathbf{u} \times \mathbf{v}$. Construct the matrix $\mathbf{u}^*$ explicitly from the components of $\mathbf{u} = \langle x, y, z \rangle$.

$$\mathbf{u}^* = \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$$

Problem 8 (3 points)

Geometrically subdivide the Bézier curve given by the control points below and at $t = 0.3$ along the curve. Label the new control points A, B, C, ..., G. (The two resulting Bézier curves should share their common point, so only 7 control points are required.)


2
4
G
C
D
B
E
1
A
F
3

Problem 9 (2 points)

Which of the following types of transformations benefit from the use of homogeneous coordinates? No explanation is necessary, but no partial credit is possible. (a) Translation, (b) uniform scale, (c) non-uniform scale, (d) rotation, (e) reflection, (f) perspective projection, (g) orthographic transformation.

(a), (f), and (g).

Problem 10 (2 points)

What is the *difference* between C^1 and G^1 continuity in the context of a car traveling down a road?

Both the G^1 car and the C^1 car are traveling down a smooth road. The speed of the C^1 driver is continuous, but the speed of the G^1 car is discontinuous.

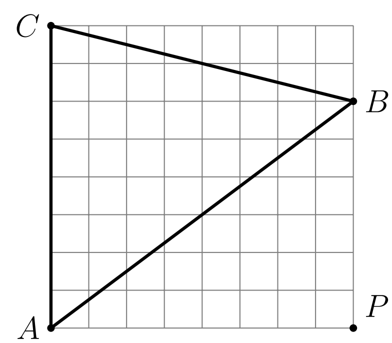
Problem 11 (2 points)

On the project, we discarded intersections closer than `small_t` when computing the closest intersection. Why?

When casting rays from a surface, such as for reflection rays and shadow rays, it is important that the ray not be considered to intersect the surface it starts on. This is done by discarding intersections closer than a tolerance `small_t`.

Problem 12 (3 points)

Find the barycentric weights for the point P in the triangle below.



To do this, we need to compute the *signed* areas of the triangles, which we can then use to compute barycentric weights. Note that all of the triangles we need have an edge that is horizontal or vertical, so $A = \frac{1}{2}bh$ is easy to compute. We must also be careful about the orientations of the triangles. Triangles that are clockwise should have negative area.

$$\text{area}(ABC) = 32 \qquad \text{area}(PBC) = 24 \qquad \text{area}(APC) = 32 \qquad \text{area}(ABP) = -24$$

$$\alpha = \frac{\text{area}(PBC)}{\text{area}(ABC)} = \frac{24}{32} = \frac{3}{4} \quad \beta = \frac{\text{area}(APC)}{\text{area}(ABC)} = \frac{32}{32} = 1 \quad \gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)} = \frac{-24}{32} = -\frac{3}{4}$$

Problem 13 (2 points)

A surface (rather like a spiral staircase) is given by the parametric equations $x = r\cos\theta$, $y = r\sin\theta$, $z = \theta$. What is the normal direction at an arbitrary point on the surface? You may express your result as a function of (r, θ) or (x, y, z) as you prefer.

$$\begin{aligned}
\mathbf{w} &= \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ \theta \end{pmatrix} \\
\mathbf{w}_r &= \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \\
\mathbf{w}_\theta &= \begin{pmatrix} -r\sin\theta \\ r\cos\theta \\ 1 \end{pmatrix} \\
\mathbf{w}_r \times \mathbf{w}_\theta &= \begin{pmatrix} (\sin\theta)1 - 0(r\cos\theta) \\ 0(-r\sin\theta) - (\cos\theta)1 \\ (\cos\theta)(r\cos\theta) - (\sin\theta)(-r\sin\theta) \end{pmatrix} = \begin{pmatrix} \sin\theta \\ -\cos\theta \\ r \end{pmatrix} \\
\|\mathbf{w}_r \times \mathbf{w}_\theta\|^2 &= (\sin\theta)^2 + (-\cos\theta)^2 + r^2 = r^2 + 1 \\
n &= \frac{\mathbf{w}_r \times \mathbf{w}_\theta}{\|\mathbf{w}_r \times \mathbf{w}_\theta\|} = \frac{1}{\sqrt{r^2+1}} \begin{pmatrix} \sin\theta \\ -\cos\theta \\ r \end{pmatrix} = \frac{1}{\sqrt{x^2+y^2+1}\sqrt{x^2+y^2}} \begin{pmatrix} y \\ -x \\ x^2+y^2 \end{pmatrix}
\end{aligned}$$

Problem 14 (3 points)

Construct a 3×3 matrix that performs each of the following 2D operations (in homogeneous coordinates). You may express it as the product of other matrices if you prefer.

(a) Rotate 45° counterclockwise about the origin.

$$\begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) Translate 2 units in the positive x direction.

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c) Rotate 90° counterclockwise about the point $(1, 2)$.

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Problem 15 (2 points)

The clipper tends to remove the significant majority of triangles. Since the vertex shader can be quite expensive, running the vertex shader after clipping could result in a significant performance boost. Nevertheless, this is never done. Why do we run the vertex shader before performing clipping?

The vertex shader transforms vertices into normalized device coordinates. Since the triangles need to be in normalized device coordinates in order to do clipping, it is not possible to clip before running the vertex shader.

Problem 16 (2 points)

Explain why (a) a flashlight shined on a nearby wall is much brighter than when shined on a far wall but (b) a laser pointer appears about the same brightness in both cases.

(a) A flashlight falls off with distance since the light rays spread out; the same amount of light energy illuminates a larger wall area. (b) Laser pointers do not spread out with distance, so the light does not fall off much. (A very small amount of light energy is lost through interactions with particles in the air.)

[1]

[1] Total points: 38