CS 130, Midterm

Solutions

Read the entire exam before beginning. **Manage your time carefully.** This exam has 35 points; you need 30 to get full credit. Additional points are extra credit. 35 points → 2.3 min/point. 30 points → 2.7 min/point.

Problem 1 (2 points)

Suggest formulas for *barycentric coordinates* for a point P relative to a tetrahedron with vertices A, B, C, and D. That is, construct formulas for weights α, β, γ, and δ such that $\alpha + \beta + \gamma + \delta = 1$ and $P = \alpha A + \beta B + \gamma C + \delta D$. You do not need to show that these properties are satisfied.

$$\alpha = \frac{\text{volume}(PBCD)}{\text{volume}(ABCD)} \quad \beta = \frac{\text{volume}(APCD)}{\text{volume}(ABCD)} \quad \gamma = \frac{\text{volume}(ABPD)}{\text{volume}(ABCD)} \quad \delta = \frac{\text{volume}(ABCP)}{\text{volume}(ABCD)}$$

Problem 2 (2 points)

Assuming that you have α, β, γ, and δ from the previous problem, suggest an algorithm for deciding whether P is inside or outside of tetrahedron $ABCD$.

The point is inside if $\alpha, \beta, \gamma, \delta \geq 0$. It is outside otherwise.

Problem 3 (2 points)

Given the triangle with vertices $A = (0,0)$, $B = (2,0)$, and $C = (0,3)$, what are the values of the barycentric weights at point (x, y)?

The barycentric weights are $\alpha = 1 - x/2 - y/3$, $\beta = x/2$, $\gamma = y/3$.

Problem 4 (2 points)

For each set of colors below, what color will be observed if a light of equal intensity of each color is shined on a white sheet of paper? No explanation is required. Write your answer in the table.

#	light colors	observed color
(a)	red ●, green ●	yellow ●
(b)	blue ●, yellow ●	white
(c)	blue ●, green ●	cyan ●
(d)	magenta ●, green ●	white

Problem 5 (2 points)

What might you see if you look at the display on your cell phone under a magnifying glass? (Assuming the phone is displaying a white background.)

You would see a pattern of red, green, and blue lights.

Problem 6 (2 points)

For each of the following four types of lights we discussed in this course, indicate whether the light falls off with distance (Y or N) and give a brief (at most one sentence) explanation for why: (a) ambient, (b) directional, (c) point, and (d) spotlight.

(a) N. Ambient light is light that is assumed to be coming from all around; moving farther from light from one source moves you closer to light from another.
(b) N. Directional lights are assumed to be so far away that the direction is essentially constant (e.g., the sun); moving a little closer or farther will make no difference.
(c) and (d) Y. This light all comes from one point and spreads out to illuminate a larger area as the distance to the source increases; this spreads out the light's energy and makes its brightness fall off with distance.

Problem 7 (5 points)

Identify the maximum and minimum possible values for each of the expressions in the table. **u** and **v** are **normalized** 3D vectors, and x is a real-valued scalar. Write $-\infty$ if there is no minimum and ∞ if there is no maximum. No justification is required, but only answers in the table count.

expression	minimum	maximum	expression	minimum	maximum
x^2	0	∞	x^3	$-\infty$	∞
$\|\mathbf{u}\|$	1	1	$\frac{1}{x^2+1}$	0	1
$\mathbf{u}\cdot\mathbf{u}$	1	1	$\mathbf{u}\cdot\mathbf{v}$	-1	1
$\|\mathbf{u}\times\mathbf{u}\|$	0	0	$\|\mathbf{u}\times\mathbf{v}\|$	0	1
$(\mathbf{u}+\mathbf{v})\cdot\mathbf{v}$	0	2	$(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v}$	0	0
$\|\mathbf{u}+\mathbf{v}\|$	0	2	$\|\mathbf{u}-\mathbf{v}\|$	0	2
$\|x\mathbf{u}\|$	0	∞	$\sin(x)$	-1	1

The remaining problems refer to the surface defined by the implicit function $f(x,y,z) = x^2 - y^2 - z$, which looks rather like a potato chip. You may assume that $f(x,y,z) > 0$ corresponds to *outside* of the surface. These problems also refer to the *ray* defined by $g(t) = \begin{pmatrix} 2t+1 \\ t+2 \\ -2t-2 \end{pmatrix}$, where $t \geq 0$. All of these problems are independent (you can solve them in any order, even if you have not solved earlier ones).

Problem 8 (2 points)

What are the direction and endpoint of the ray?

Endpoint is $g(0) = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$. Direction is $\frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$. (Don't forget to normalize!)

Problem 9 (2 points)

Compute all intersection *locations* of the surface with the ray. [Hint: the answer should come out reasonably nicely; if it does not, check your math.]

Plugging $g(t)$ into $f(x,y,z)$ for the surface we get

$$\begin{aligned} 0 &= x^2 - y^2 - z \\ &= (2t+1)^2 - (t+2)^2 - (-2t-2) \\ &= (4t^2 + 4t + 1) - (t^2 + 4t + 4) + 2t + 2 \\ &= 3t^2 + 2t - 1 \\ &= (3t-1)(t+1) \\ t &= -1, \frac{1}{3} \end{aligned}$$

Since this is a ray and $t \geq 0$, we can exclude the first. This leaves $t = \frac{1}{3}$, which corresponds to $g(3) = \begin{pmatrix} \frac{5}{3} \\ \frac{7}{3} \\ -\frac{8}{3} \end{pmatrix}$.

Problem 10 (2 points)

What is the normal direction at an arbitrary point (x, y, z) lying on the surface of the surface? Don't worry about whether the normal points inwards or outwards.

$$f = x^2 - y^2 - z$$

$$\nabla f = \begin{pmatrix} 2x \\ -2y \\ -1 \end{pmatrix}$$

$$\|\nabla f\| = \sqrt{4x^2 + 4y^2 + 1}$$

$$n = \frac{\nabla f}{\|\nabla f\|} = \frac{1}{\sqrt{4x^2 + 4y^2 + 1}} \begin{pmatrix} 2x \\ -2y \\ -1 \end{pmatrix}$$

The normal direction can also be worked out using a parametric representation such as $g(x, y) = (x, y, x^2 - y^2)$. In this case,

$$\frac{\partial g}{\partial x} \times \frac{\partial g}{\partial y} = \begin{pmatrix} 1 \\ 0 \\ 2x \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ -2y \end{pmatrix} = \begin{pmatrix} -2x \\ 2y \\ 1 \end{pmatrix}$$

Normalizing leads to the same result, up to sign.

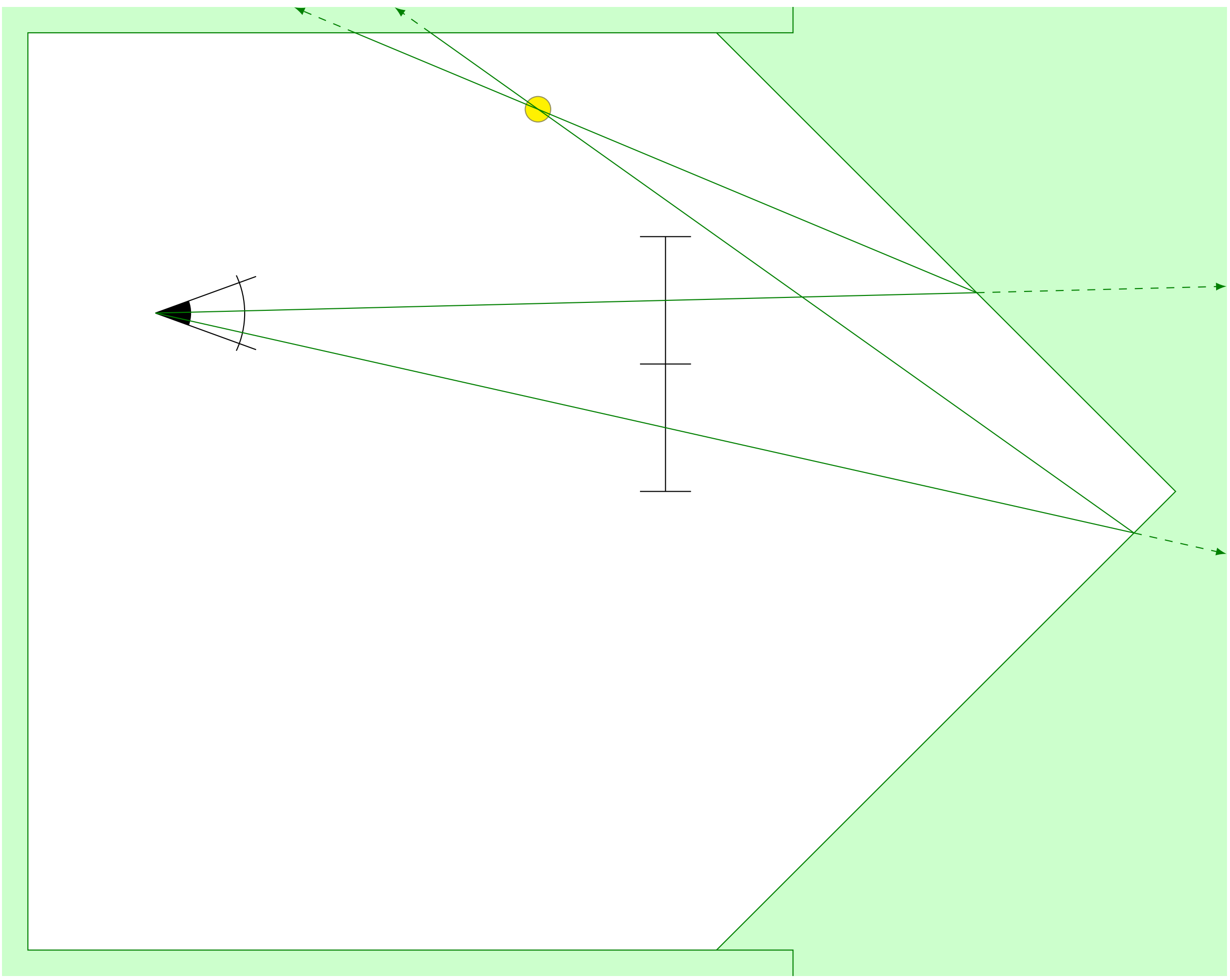
Problem 11 (2 points)

Let $\mathbf{z} = \mathbf{e} + t\mathbf{u}$, where $\mathbf{z}, \mathbf{e}, \mathbf{u}$ are vectors and are given to you. You may assume that $\mathbf{u}$ has been correctly normalized. Find t. *Be careful.*

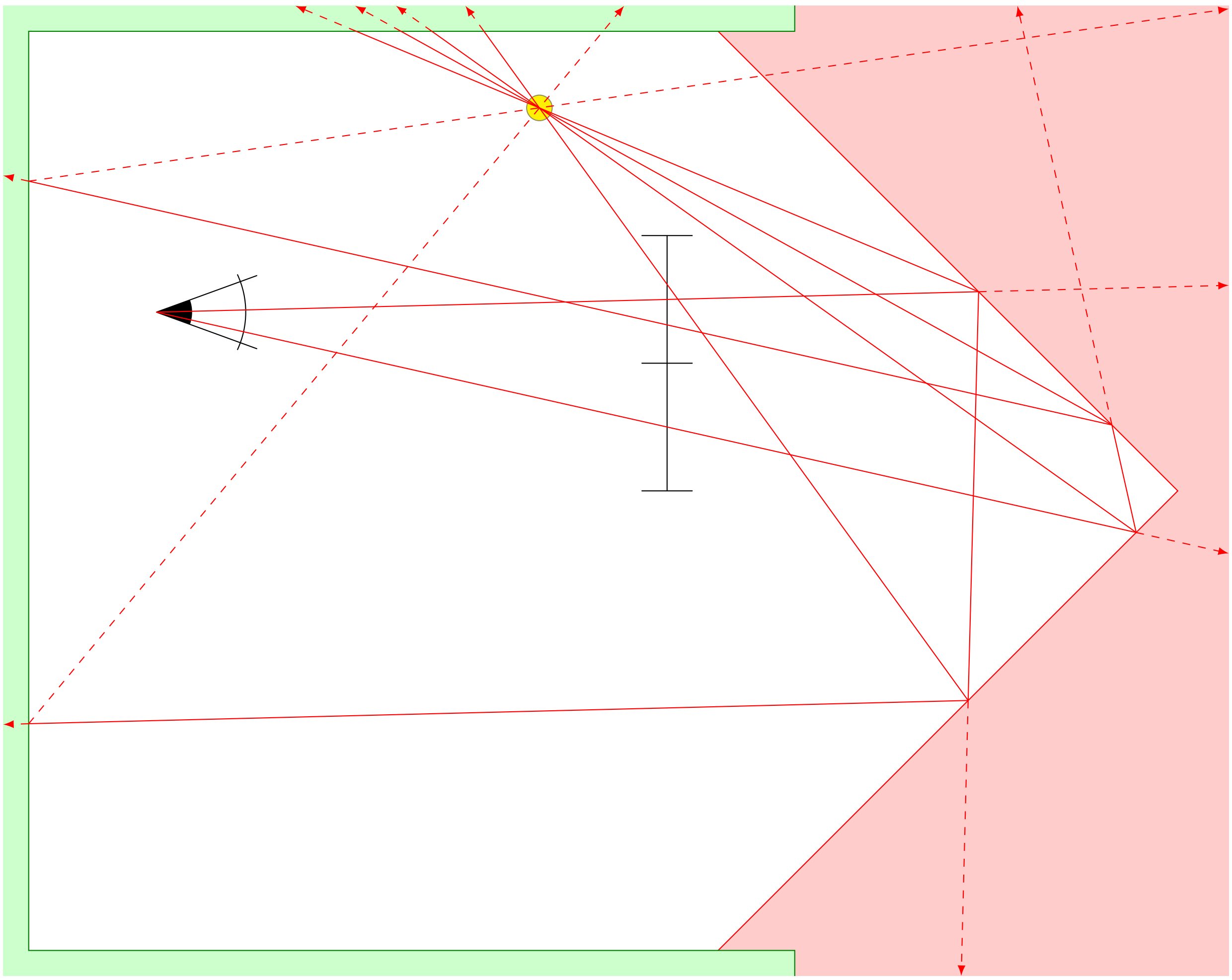
Take the dot product by $\mathbf{u}$ and solve, which yields $t = (\mathbf{z} - \mathbf{e}) \cdot \mathbf{u}$.

In the raytracing problems below, green objects are wood, red objects are reflective, and blue objects are transparent. The scenes are in 2D with a 1D image. yellow circles are point lights; the ray tracer supports shadows. Draw all of the rays that would be cast while raytracing each scene. Use a maximum recursion depth of 6. (Don't worry about precisely what counts as depth 6; I just care that recursion is being performed correctly when necessary and that important rays are not missing. There are no more than 25 rays in the "exact" solution.)

Problem 12 (4 points)



Problem 13 (4 points)



Problem 14 (2 points)

Find the normal direction for the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$

$$N = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$n = \frac{N}{\|N\|} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

[1]

[1]Total points: 35