CS 130, Midterm

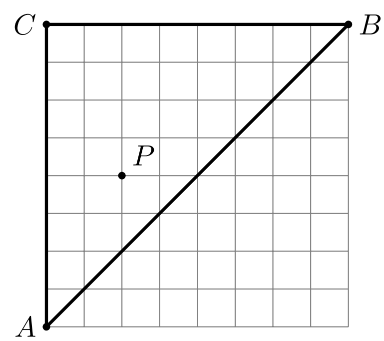
Solutions

1	2	3	4	5	6	7	8	$\sum$

Read the entire exam before beginning. **Manage your time carefully.** This exam has 34 points; you need 26 to get full credit. Additional points are extra credit. 34 points → 1.5 min/point. 26 points → 1.9 min/point.

Problem 1 (3 points)

Find the barycentric weights for the point P in the triangle below.



To do this, we need to compute the areas of the triangles, which we can then use to compute barycentric weights. Note that all but one of the triangles we need have an edge that is horizontal or vertical, so $A = \frac{1}{2}bh$ is easy to compute.

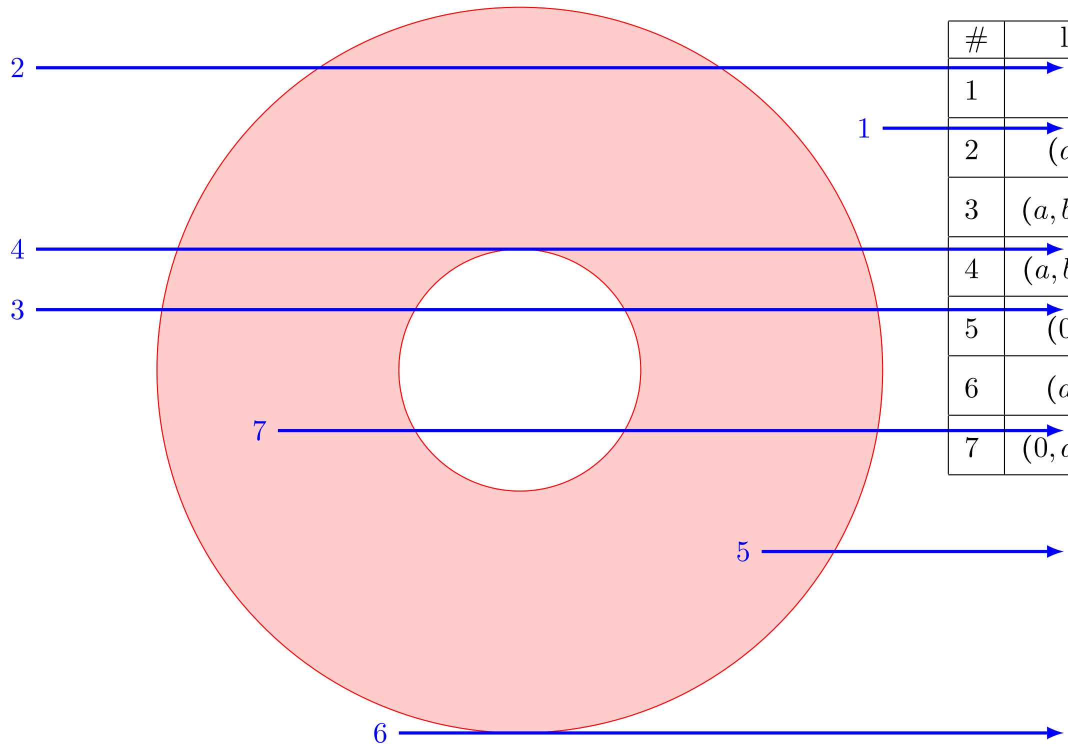
$$\text{area}(ABC) = 32 \qquad \text{area}(PBC) = 16 \qquad \text{area}(APC) = 8$$

$$\text{area}(ABP) = \text{area}(ABC) - \text{area}(PBC) - \text{area}(APC) = 8$$

$$\alpha = \frac{\text{area}(PBC)}{\text{area}(ABC)} = \frac{16}{32} = \frac{1}{2} \qquad \beta = \frac{\text{area}(APC)}{\text{area}(ABC)} = \frac{8}{32} = \frac{1}{4} \qquad \gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)} = \frac{8}{32} = \frac{1}{4}$$

Problem 2 (3 points)

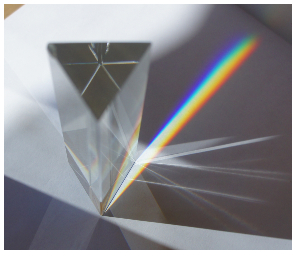
When raytracing Booleans, we must keep track of a list of intersection distances along the ray. When intersecting with the object below, many different lists are possible. For each of the intersection lists in the table, draw and label (1,2,...) a ray what would produce such an intersection list.



#	list
1	$()$
2	(a, b)
3	(a, b, c, d)
4	(a, b, b, c)
5	$(0, a)$
6	(a, a)
7	$(0, a, b, c)$

Problem 3 (2 points)

A prism is a transparent object that has different indexes of refraction for different wavelengths of light. As a result, for example, red and yellow light bend by different angles as they pass through the prism. In this way, sunlight is divided into a rainbow upon passing through a prism. (See the photograph to the right.) What would be observed if light from a computer screen displaying a solid white background were passed through the prism instead of white light from the sun?



Instead of a rainbow, you might see colored bands for red, green, and blue (in the same places that they occur in the rainbow) with no light between those bands. If the red, green, and blue are not pure, a few bands are observed. A rainbow will not be observed. (For example, sunlight contains light that is actually yellow. Yellow in a computer monitor is red plus green.)

Problem 4 (4 points)

Compute the normal direction for the implicit surface $f(x, y) = 2x^2y^2 - 5xy^2 + 2$ at the point $(2, -1)$. You do not need to simplify your answer.

$$\begin{aligned}
\mathbf{u} = \nabla f &= \begin{pmatrix} 4xy^2 - 5y^2 \\ 4x^2y - 10xy \end{pmatrix} = \begin{pmatrix} 4(2)(-1)^2 - 5(-1)^2 \\ 4(2)^2(-1) - 10(2)(-1) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \\
\|\mathbf{u}\| &= 5 \\
\mathbf{n} &= \frac{\mathbf{u}}{\|\mathbf{u}\|} = \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}
\end{aligned}$$

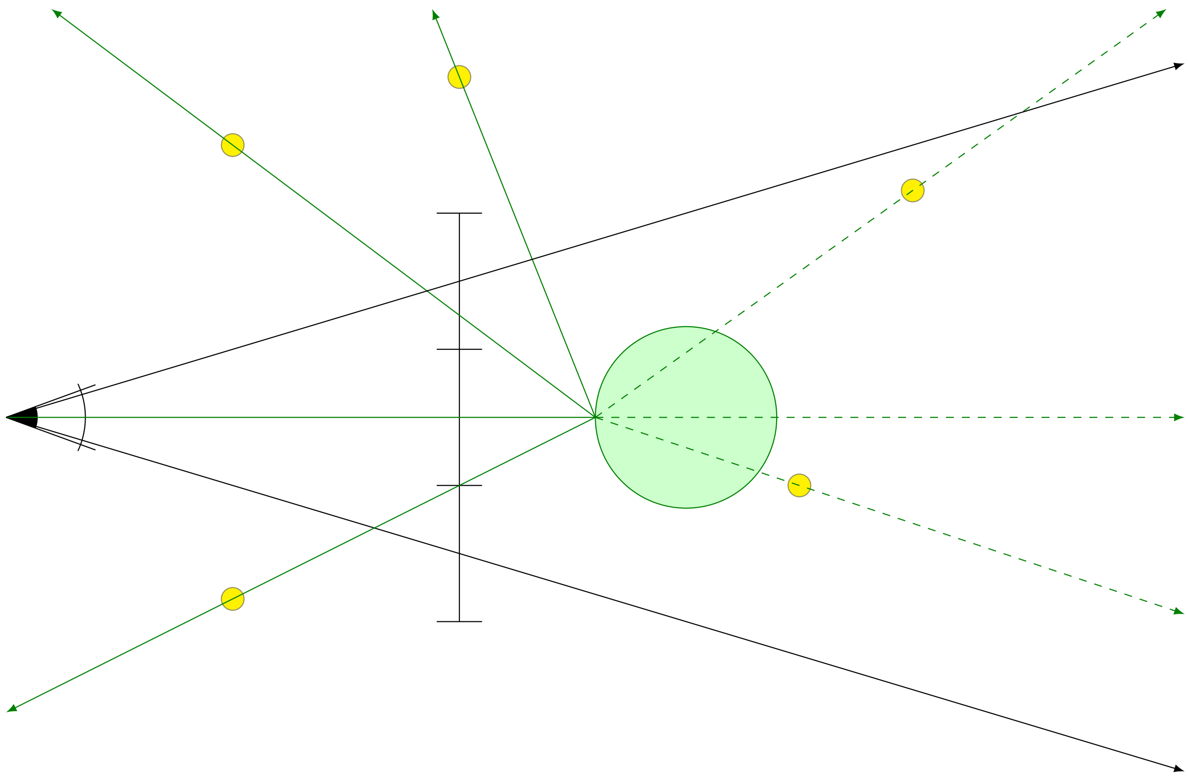
Problem 5 (4 points)

Find the intersection between a ray (endpoint $\mathbf{p}$, direction $\mathbf{u}$) and a plane (point $\mathbf{z}$, normal $\mathbf{n}$). You may assume that there is exactly one intersection.

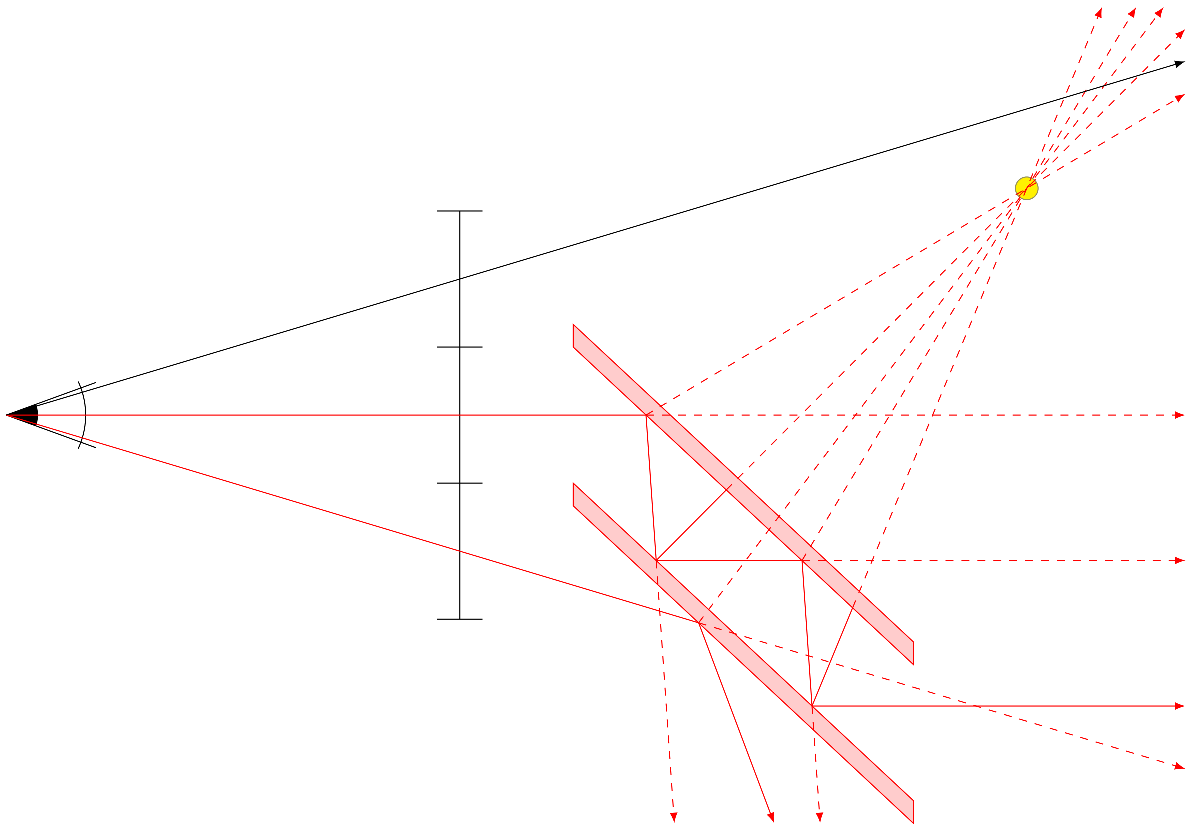
$$\begin{aligned}
\mathbf{x} &= \mathbf{p} + t\mathbf{u} \\
(\mathbf{x} - \mathbf{z}) \cdot \mathbf{n} &= 0 \\
(\mathbf{p} + t\mathbf{u} - \mathbf{z}) \cdot \mathbf{n} &= 0 \\
t(\mathbf{u} \cdot \mathbf{n}) &= (\mathbf{z} - \mathbf{p}) \cdot \mathbf{n} \\
t &= \frac{(\mathbf{z} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{u} \cdot \mathbf{n}} \\
\mathbf{x} &= \mathbf{p} + \frac{(\mathbf{z} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{u} \cdot \mathbf{n}} \mathbf{u}
\end{aligned}$$

In the raytracing problems below, green objects are wood, red objects are reflective, and blue objects are transparent. The scenes are in 2D with a 1D image. yellow circles are point lights; the ray tracer supports shadows. Draw all of the rays that would be cast while raytracing each scene. Use a maximum recursion depth of 3. (Don't worry about precisely what counts as depth 3 or depth 4; I just care that recursion is being performed correctly when necessary and that important rays are not missing. There are no more than 20 rays in the "exact" solution.)

Problem 6 (6 points)



Problem 7 (6 points)



Problem 8 (6 points)