$m\ddot{x} = +f = -\nabla\phi \Rightarrow m_1\ddot{x}_1 = f_1 = \frac{\partial\phi}{\partial x_1} \quad m_2\ddot{x}_2 = f_2 = \frac{\partial\phi}{\partial x_2}$

$\phi = g(x_1 - x_2) \Rightarrow \frac{\partial\phi}{\partial x_1} = \nabla g(x_1 - x_2) \quad \frac{\partial\phi}{\partial x_2} = -\nabla g(x_1 - x_2)$ chain rule.

$\Rightarrow f_1 = -f_2$ $\boxed{\text{shift invariance}}$

$m_1\ddot{x}_1 + m_2\ddot{x}_2 = f_1 + f_2 = 0$

$m_1\dot{x}_1 + m_2\dot{x}_2 = \text{const}$ conservation of momentum

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$\phi(x_1 - x_2) = h(\|x_1 - x_2\|)$ $\boxed{\text{rotation invariance}}$

$\ell = \|x_1 - x_2\| = \|\omega\| \quad \omega = x_1 - x_2 \quad \hat{\omega} = \frac{\omega}{\ell}$

$2\ell\frac{\partial\ell}{\partial\omega} = 2\omega \quad \ell^2 = \omega\cdot\omega$

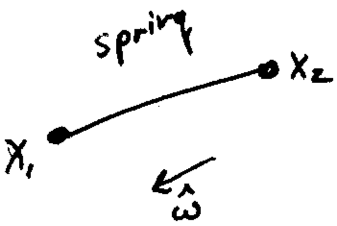
$\frac{\partial\ell}{\partial\omega} = \hat{\omega}$

$\frac{\partial\phi}{\partial x_1} = \frac{\partial}{\partial x_1}h(\ell) = h'(\ell)\frac{\partial\ell}{\partial x_1} = h'(\ell)\hat{\omega}^T\frac{\partial\omega}{\partial x_1} = h'(\ell)\hat{\omega}$

$\frac{\partial\phi}{\partial x_2} = h'(\ell)\hat{\omega}^T\frac{\partial\omega}{\partial x_2} = -h'(\ell)\hat{\omega}$

$$\boxed{\begin{aligned} m_1\ddot{x}_1 &= f_1 = -h'(\ell)\hat{\omega} \\ m_2\ddot{x}_2 &= -f_1 = f_2 = h'(\ell)\hat{\omega} \end{aligned}}$$

$\hat{\omega}$ is direction from x_2 to x_1



$$-f_2 = f_1 = -h'(\ell)\,\hat{\omega}$$

* if $\ell = \ell_0 \Rightarrow f_1 = 0$ $\qquad$ ℓ_0 is restlength

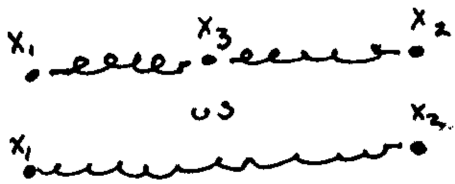
if $\ell > \ell_0$ stretched $\Rightarrow f_1$ opposite $\hat{\omega}$

$-h'(\ell)$ is neg

$\boxed{h'(\ell) \text{ is pos}}$

$$h'(\ell) = \frac{k}{\ell_0}(\ell - \ell_0)$$

normalize.



same force on f_1

but ℓ and ℓ_0 are doubled

$\rightarrow$ normalize for ℓ_0

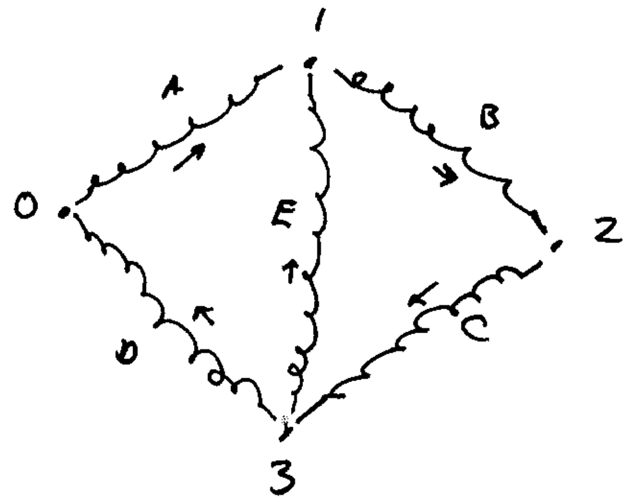
integrate: $\quad h(\ell) = \frac{1}{2}\frac{k}{\ell_0}(\ell - \ell_0)^2$

note: customary to have $h(\ell)$ have min 0. can <u>shift</u> $h(\ell)$.

only $h'(\ell)$ actually matters

$$\phi = \frac{k}{2\ell_0}(\ell - \ell_0)^2 = \frac{k\ell_0}{2}\left(\frac{\ell}{\ell_0} - 1\right)^2$$

$$\boxed{f_1 = -k\left(\frac{\ell}{\ell_0} - 1\right)\frac{x_1 - x_2}{\|x_1 - x_2\|} = -f_2}$$



$M_0 \ddot{X}_0 = -f_A + f_D$

$M_1 \ddot{X}_1 = f_A + f_E - f_B$ 12 degrees of freedom

$M_2 \ddot{X}_2 = f_B - f_C$

$M_3 \ddot{X}_3 = f_C - f_D - f_E$

note: k_A k_B ... could be different strengths.